# CS551: Introduction to Deep Learning <br> Mid Semester, Spring 2018 <br> IIT Patna 

## Attempt all questions. Do not write anything on the question paper.

Time: 2 Hrs
Full marks: 30

1. Prove or disprove: $\operatorname{tr}(A B)=\operatorname{tr}(A) \times \operatorname{tr}(B)$ where $A$ and $B$ are $n \times n$ matrices and $\operatorname{tr}$ denotes the trace.
2. A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a (false) positive when applied to a non-sufferer. It is estimated that $0.5 \%$ of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). Calculate the following probabilities: (a) that the test result will be positive; (b) that, given a positive result, the person is a sufferer; (c) that, given a negative result, the person is a non-sufferer; (d) that the person will be misclassified.
$(1+1+2+2)$
3. Consider a neural network where the last output layer uses softmax as activation function for some classification problem. Following is an observation on three training examples where $o_{i}$ denotes the probability of each class from softmax and $t_{i}$ denotes the labeled target for the examples. Find out mean cross entropy error for these three examples.

| $o_{1}$ | $o_{2}$ | $o_{3}$ | $t_{1}$ | $t_{2}$ | $t_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.3 | 0.6 | 0 | 0 | 1 |
| 0.2 | 0.6 | 0.2 | 0 | 1 | 0 |
| 0.3 | 0.4 | 0.3 | 1 | 0 | 0 |

4. Consider maximal margin classifier for the following toy data set which has two features $X_{1}$ and $X_{2} . Y$ is the target label . (a) Sketch the optimal separating hyperplane, and provide the equation for this hyperplane. [Equation for hyperplane may be derived logically or mathematically.] (b) On your sketch, indicate the margin for the maximal margin hyperplane. (c) Indicate the support vectors for the maximal margin classifier.

| Sl. No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | 3 | 2 | 4 | 1 | 2 | 4 | 4 |
| $X_{2}$ | 4 | 2 | 4 | 4 | 1 | 3 | 1 |
| $Y$ | R | R | R | R | B | B | B |

5. Given a set of $m$ points $\left\{\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(m)}\right\}$ in $\mathbb{R}^{n}$ and we want represent these points in $k$ dimension where $k<n$. Propose a suitable methodology for it.
