Introduction to Deep Learning

Arijit Mondal
Dept. of Computer Science & Engineering
Indian Institute of Technology Patna
arijit@iitp.ac.in

Deep Reinforcement Learning

Introduction

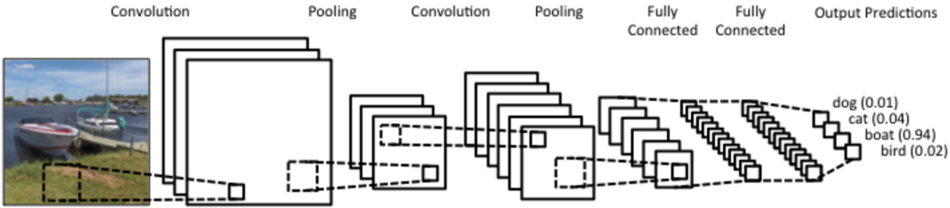
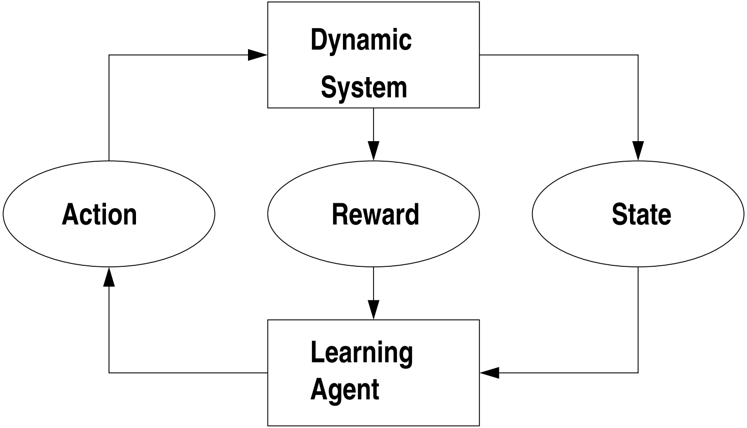


Image source: http://www.wildml.com/2015/11/understanding-convolutional-neural-networks-for-nlp/

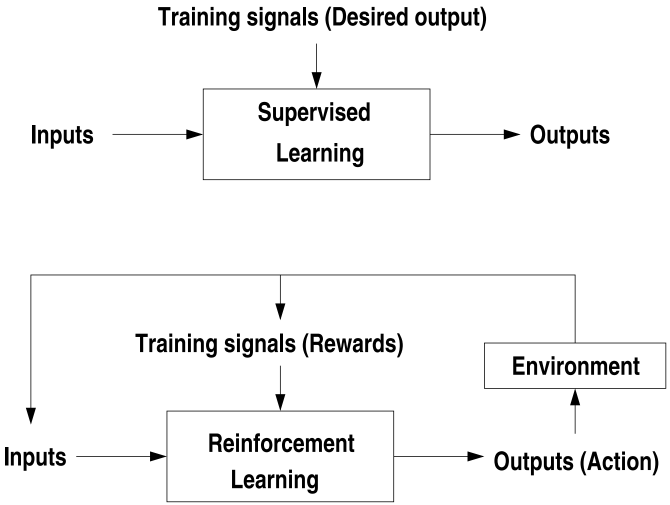
Interaction with environment



Reinforcement learning

- **Set of actions that the learner will make in order to maximize its profit**
- **Action may not only affect the next situation but also subsequent situation**
  - **Trial and error search**
  - **Delayed reward**
- **A learning agent is interacting with environment to achieve a goal**
- **Agent needs to have idea of state so that it can take right action**
- **Three key aspects — observation, action, goal**

Reinforcement vs supervised learning



Reinforcement learning

- **It is different from supervised learning**
  - **Learning from examples provided by a knowledgeable external supervisor**
  - **Not adequate for learning from interaction**
- **In interaction problem it is often impractical to obtain examples of desired behavior that are correct and representative of all situations**
- **Trade-off between** *exploration* **and** *exploitation*
  - **To improve reward it must prefer effective action from the past (exploit)**
  - **To discover such action it has to try unselected actions (explore)**
  - **Exploit and exploration cannot be pursued exclusively**
- **Agent interacts with uncertain environment**

When to use RL

- **Data in the form of trajectories**
- **Need to make a sequence of decision**
- **Observe (partial, noisy) feedback to state or choice of action**

Examples

- Chess player eg. games
- Robotics
- Adaptive controller
- All involve interaction between active decision making agent and its environment

Elements of RL

- Agent
- Environment
- Policy — The way agent behaves at a given time
  - Mapping of state-action pair to state
  - Can use look up table or search method
  - Core of reinforcement learning problem
- Reward function — Defines the goal in reinforcement learning problem
  - It maps state-action pair to a single number
  - Objective of RL agent is to maximize total reward
  - Defines bad or good events
  - Must be unalterable by agent, however policy can be changed

Elements of RL (contd.)

- **Value function**
  - **Specifies what is good in long run**
  - **Value of a state is the total amount of reward an agent can expect to accumulate over future starting from the state**
  - **Indicates long term desirability of states**
  - **The action tries to move to a state of highest value (not highest reward)**
  - **Rewards are mostly given by the environment**
  - **Value must be estimated or reestimated from the sequence of observation**
  - **Need efficient method to find values**
    - **Evolutionary methods (genetic algorithm, simulated annealing) search directly in the space of policies without applying value function**

Elements of RL (contd.)

- **Model of environment**
  - **Mimics the behavior of environment**
  - **Given state and action, model might predict resultant next state and next reward**
  - **Every RL system uses trial and search methodology to learn**

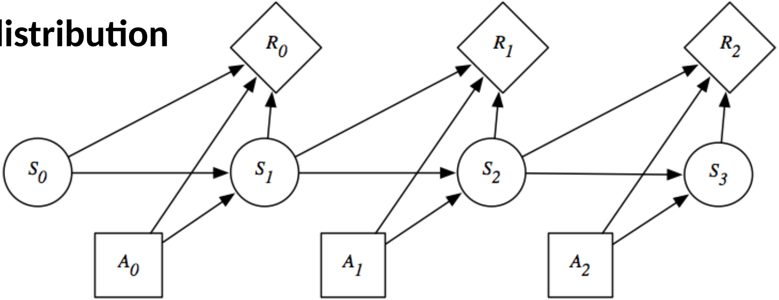
Reinforcement learning

- **Learning agent tries a sequence of actions (a_t)**
- **Observes outcomes (state s_{t+1}, rewards r_t) of those actions**
- **Statistically estimated relationship between action choice and outcomes $Pr(s_t|s_{t-1}, a_{t-1})$**
- **Selection of policy $\pi(s)$ that optimizes selected outcome**

$$\arg\max_{\pi} E_{\pi}[r_0 + r_1 + \ldots + r_T | s_0]$$

Markovian decision process

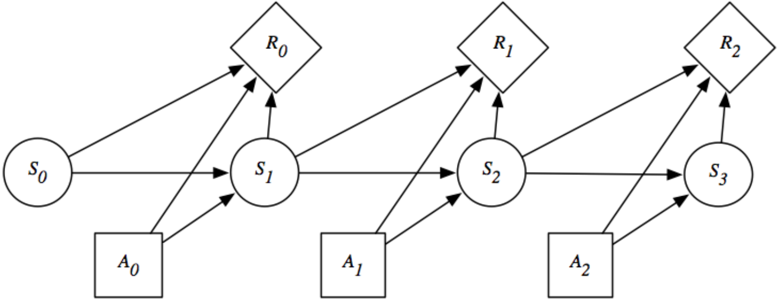
- S — set of states
- A — set of actions
- $Pr(s_t | s_{t-1}, a_{t-1})$ — Probabilistic effects
- r_t — reward function
- μ_t — initial state distribution



The Markov property

- **The future state depends only on the current state**

$$Pr(s_t | s_{t-1}, \ldots, s_0) = Pr(s_t | s_{t-1})$$



Utility maximization

- Let U_t be the utility for a trajectory starting from t
- Episodic tasks (eg. games)

$$U_t = r_t + r_{t+1} + r_{t+2} + \ldots + r_T$$

- Continuing tasks (eg. can run forever)

$$U_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots = \sum_{k=0} \gamma^k r_{t+k}$$

- γ is known as discount factor and lies between 0 and 1
  - At each time step there is a chance of $(1-\gamma)$ that agent dies and no reward after that
  - Inflation rate - receiving an amount of money today, the value of it tomorrow will be less by a factor of γ

Policy

- **Policy defines the action selection strategy at every state**

$$\pi(s,a) = P(a_t = a, s_t = s)$$

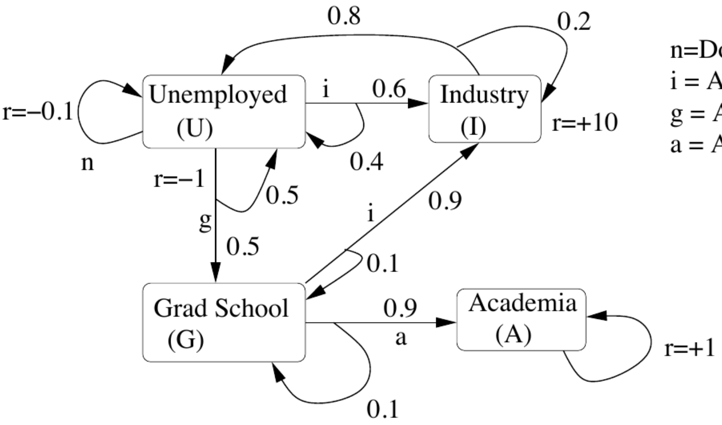
  - **It can be stochastic or deterministic**

- **Goal is to maximize expected total reward**

$$\arg\max_{\pi} E_{\pi}[r_0 + r_1 + \ldots + r_T | s_0]$$

  - **There are many policies!**

Example



n=Do Nothing
i = Apply to industry
g = Apply to grad school
a = Apply to academia

Image source: Reinforcement Learning by Joelle Pineau

Value functions

- As we are looking for best policy, it will be useful to estimate the expected return
- Good policy may be chosen by searching over the space of policies
- Value function at a state under a given policy is

$$V^{\pi}(s) = E_{\pi}[r_t + r_{t+1} + \ldots + r_T | s_t = s]$$

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\begin{aligned}
V^{\pi}(s) &= E_{\pi}[r_t + r_{t+1} + \ldots + r_T | s_t = s] \\
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V^{\pi}(s) &= \sum_{a \in A} \pi(s,a) r(s,a) + E_{\pi}[r_{t+1} + \ldots + r_T | s_t = s]
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\end{aligned}
$$

Value of policy

- **Reorganizing the last expression**

$$V^{\pi}(s) = \sum_{a \in A} \pi(s,a) \left(r(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^{\pi}(s') \right)$$

Value of policy

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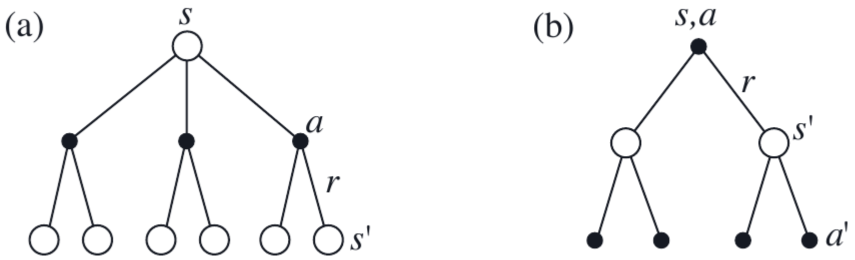
$$V^{\pi}(s) \;=\; \sum_{a \in A} \pi(s,a)\left(r(s,a) + \gamma \sum_{s' \in S} T(s,a,s')V^{\pi}(s')\right)$$

- **If we have state-action value functions**

$$Q^{\pi}(s,a) \;=\; r(s,a) + \gamma \sum_{s' \in S} P(s'|a,s) \sum_{a'} \pi(s',a')Q^{\pi}(s',a')$$

- **Known as Bellman's equation**

Value computation



Value of policy

- State value function

$$V^{\pi}(s) \;=\; \sum_{a \in A} \pi(s,a) \left(r(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^{\pi}(s') \right)$$

- In case of finite number of states, we have a system of linear equations with unique solution to V^{π}
- Above equation can be written in matrix form as

$$\boldsymbol{V}^{\pi} = \boldsymbol{R}^{\pi} + \gamma \boldsymbol{T}^{\pi} \boldsymbol{V}^{\pi}$$

- Solution will be

$$\boldsymbol{V}^{\pi} = (1 - \gamma \boldsymbol{T}^{\pi})^{-1} \boldsymbol{R}^{\pi}$$

Iterative policy evaluation

- Guess initial values for $V_0(s)$
  - It can be 0
- In every iteration say k, the value function for every state will be updated as

$$V_{k+1} = R(s, \pi(s)) + \gamma \sum_{s'} T(s, \pi(s), s') V_k(s')$$

- Iteration will stop when the difference between two consecutive iteration is within a given threshold

Convergence of iterative policy evaluation

- **Absolute error in after $(k+1)$th iteration**

$$
\begin{aligned}
V_{k+1}(s) - V^{\pi}(s) &= |\sum_{a} \pi(s,a)(R(s,a) + \gamma \sum_{s'} T(s,a,s')V_k(s') \\
&\quad - \sum_{a} \pi(s,a)(R(s,a) - \gamma \sum_{s'} T(s,a,s')V^{\pi}(s')| \\
&\leq \gamma \sum_{a} \pi(s,a) \sum_{s'} T(s,a,s')|V_k(s') - V^{\pi}(s')|
\end{aligned}
$$

- **If $\gamma \leq 1$, then error reduces to 0 gradually**

Optimal value function

- **Optimal value function may be defined as**

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

- **Any policy that achieves the optimal value function is known as optimal policy**
  - **Usually denoted as π^***
- **Optimal value is unique**
- **Optimal policy is not necessarily unique**

Optimal value function

- **Suppose V^*, R, T, γ are known, then π^* can be determined as**

$$\pi^*(s) = \arg\max_{a \in A} \left(r(s,a) + \gamma \sum_{s'} T(s,a,s') V^*(s) \right)$$

Optimal value function

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$$\pi^*(s) = \arg\max_{a \in A} \left(r(s,a) + \gamma \sum_{s'} T(s,a,s') V^*(s) \right)$$

- **Suppose π^*, R, T, γ are known, then V^* can be determined as**

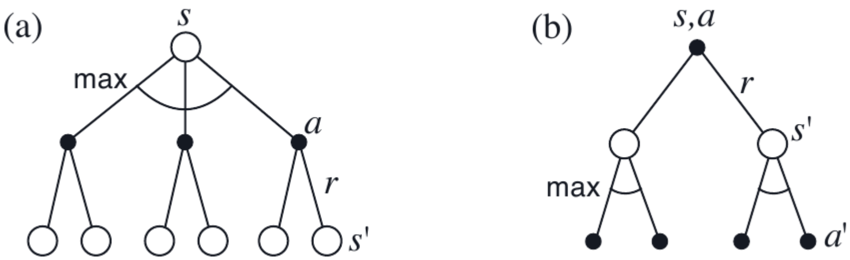
$$\begin{aligned} V^*(s) &= \sum_{a \in A} \pi^*(s,a) \left(r(s,a) + \gamma \sum_{s'} T(s,a,s') V^*(s) \right) \\ V^*(s) &= r(s,\pi(s)) + \gamma \sum_{s'} T(s,\pi(s),s') V^*(s') \end{aligned}$$

Optimal value function

- **For state-action pair**

$$Q^*(s,a) \;=\; r(s,a) + \gamma \sum_{s' \in S} P(s'|a,s) \max_{a'} Q^*(s',a')$$

Optimal value computation



Recycling Robot

- A robot does one of the following at each time step
  - Actively search for a can
  - Remain stationary and wait for someone to bring a can
  - Go back to home base to recharge battery

Recycling Robot: Transition relation

s	s'	a	$p(s'\|s,a)$	$r(s,a,s')$
high	high	search	α	r_{search}
high	low	search	$1-\alpha$	r_{search}
low	high	search	$1-\beta$	-3
low	low	search	β	r_{search}
high	high	wait	1	r_{wait}
high	low	wait	0	r_{wait}
low	high	wait	0	r_{wait}
low	low	wait	1	r_{wait}
low	high	recharge	1	0
low	low	recharge	0	0

Example

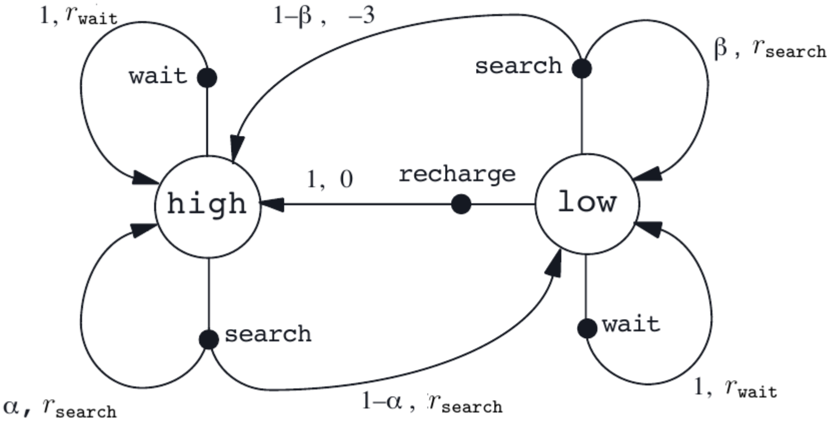


Image source: Reinforcement Learning by Andrew Barto and Richard S. Sutton

Optimal value computation

- **For recycling robot**

$$
\begin{aligned}
V^*(h) &= \max\left\{\begin{array}{l} p(h|h,s)[r(h,s,h)+\gamma V^*(h)] + p(l|h,s)[r(h,s,l)+\gamma V^*(l)], \\ p(h|h,w)[r(h,w,h)+\gamma V^*(h)] + p(l|h,w)[r(h,w,l)+\gamma V^*(l)] \end{array}\right\} \\
V^*(h) &= \max\{r_s + \gamma[\alpha V^*(h) + (1-\alpha)V^*(l)],\ r_w + \gamma V^*(h)\}
\end{aligned}
$$

$$
V^*(l) = \max\left\{\begin{array}{l} \beta r_s - 3(1-\beta) + \gamma[(1-\beta)V^*(h) + \beta V^*(l)] \\ r_w + \gamma V^*(l), \\ \gamma V^*(h) \end{array}\right\}
$$

Finding a good policy (iterative approach)

- **Start with an initial policy π_0**
- **Repeat the following**
  - **Determine the V^{π} using policy evaluation**
  - **Determine a new policy π' which is greedy with respect to V^{π}**
- **Terminate when $\pi = \pi'$**

Finding a good policy (iterative approach)

- **Start with an initial value $V_0(s)$**
- **In every iteration, update the value function**

$$V_k(s) = \max_{a \in A}\left(R(s,a) + \gamma \sum_{s'} T(s,a,s')V_{k-1}(s')\right)$$

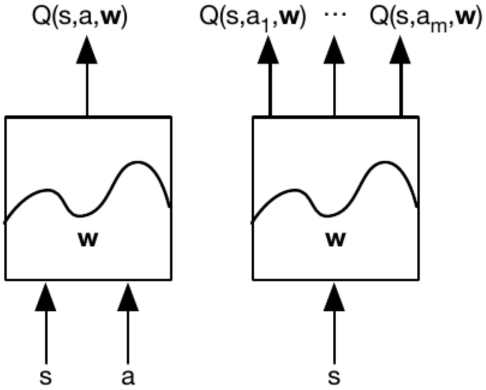
- **Stop when maximum value change between iterations is below threshold**
- **The algorithm converges to the true value of V^***

Approaches to RL

- **Value based RL**
  - Estimate the optimal value function $Q^*(s, a)$
  - The maximum value that can be achieved under any policy
- **Policy based RL**
  - Look for optimal policy π^*
  - Policy achieving maximum future reward
- **Model based RL**
  - A model of the environment is developed
  - Plan is made using the model

Q-Networks

- **Represent value function by Q-network with weights $\mathbf{w}$, $Q(s, a, \mathbf{w}) \approx Q^*(s, a)$**



Q learning

- **Optimal Q-values should obey Bellman equation**

$$Q^*(s,a) \quad = \quad E_{s'}\left[r + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

- **Right hand side may be treated as target**
- **Minimize MSE loss by SGD**

$$l \quad = \quad \left(r + \gamma \max_{a'} Q(s',a',\mathbf{w}) - Q(s,a,\mathbf{w})\right)^2$$

- **Can diverge using neural networks because of**
  - **Correlations between samples**
  - **Non-stationary targets**

Deep Q network

- **Data set are generated from agents own experience**

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

- **Sample experience from data set and apply update**

$$I = \left(r + \gamma \max_{a'} Q(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w})\right)^2$$

Deep reinforcement learning

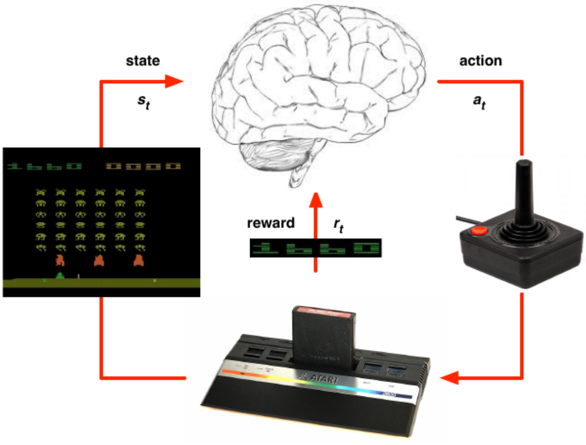


Image source:Deep reinforcement learning by David Silver

DQN

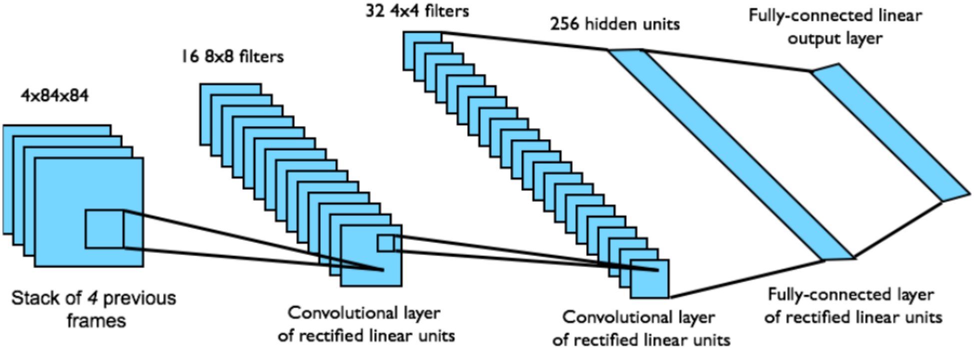


Image source:Deep reinforcement learning by David Silver

References

- *Reinforcement Learning: An Introduction* **by Andrew Barto and Richard S. Sutton**
- *Human-level control through deep reinforcement learning* **by Deep Mind, Google**