

# Modeling: Hybrid systems



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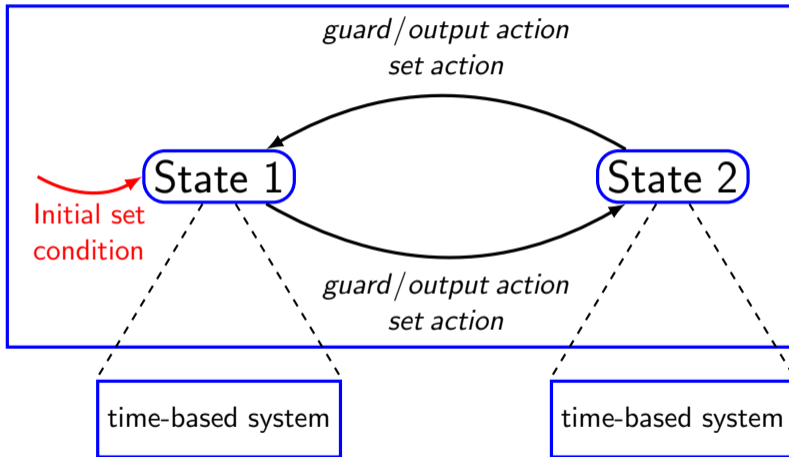
# Introduction

- Most of the systems have both continuous and discrete behavior
- Continuous behavior can be modeled by ODE and discrete behavior by FSM
- Need to have separate modeling scheme to describe both the behavior
- Usually, states in discrete modelling are enhanced with time based behavior

# Hybrid systems: example

- Digital controller
  - Thermostat
  - Automatic cruise control
  - Aircraft autopilot
- Phased operation
  - Bouncing ball
  - Biological cell growth
- Multiagent systems
  - Interaction of robots
  - Ground and air transportation systems

# Timed automata

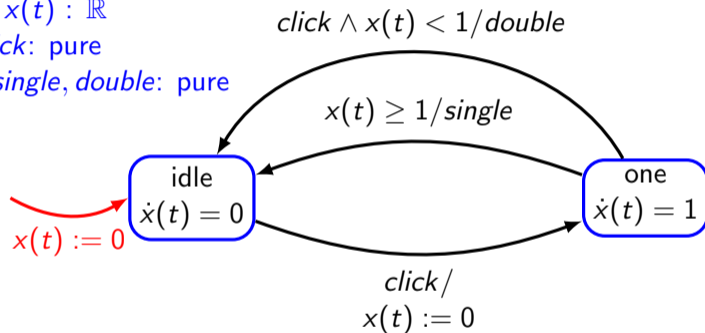


# Double click detector

**variable:**  $x(t) : \mathbb{R}$

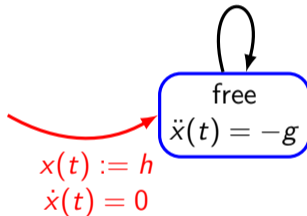
**input:**  $click$ : pure

**output:**  $single, double$ : pure

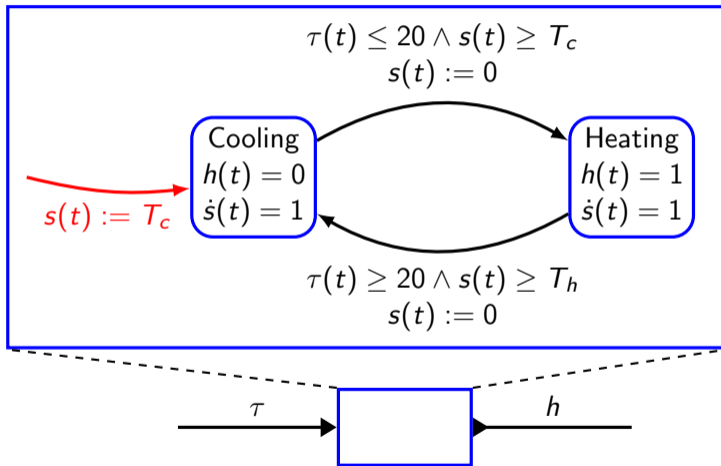


# Bouncing ball

$$x(t) = 0 / \text{bump}$$
$$\dot{x}(t) := -a\dot{x}(t)$$



# Thermostat

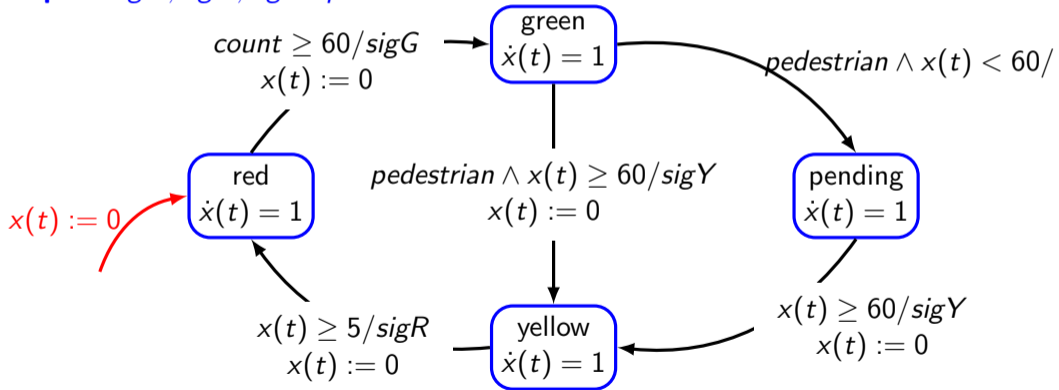


# Example: pedestrian crosswalk

**variable:**  $count : \{0, 1, \dots, 60\}$

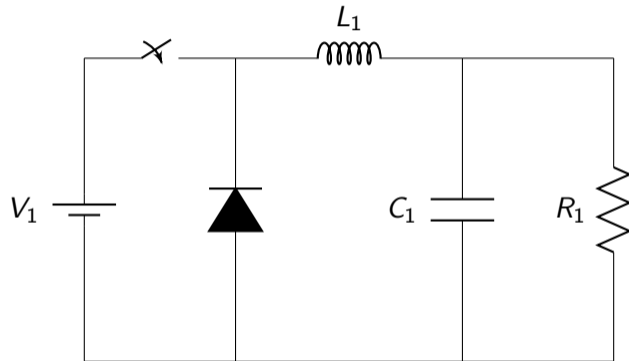
**input:**  $pedestrian : pure$

**output:**  $sigY, sigG, sigR : pure$

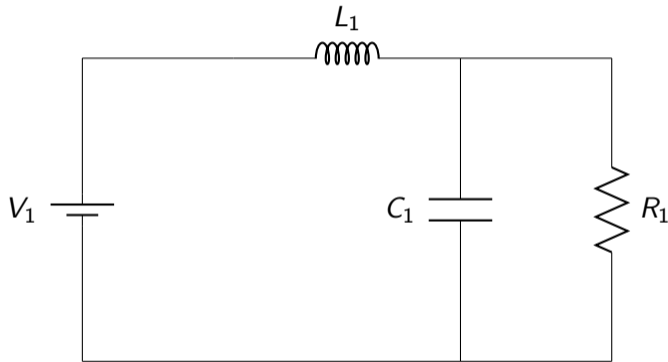




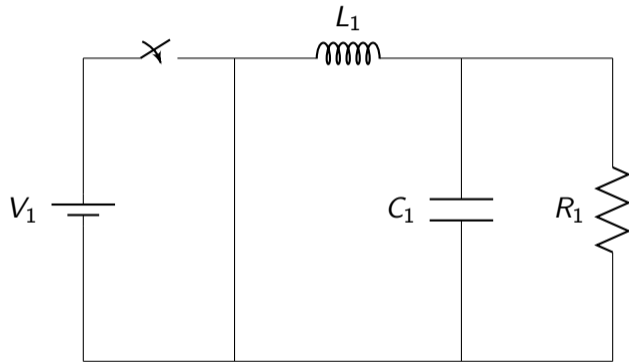
# Buck converter



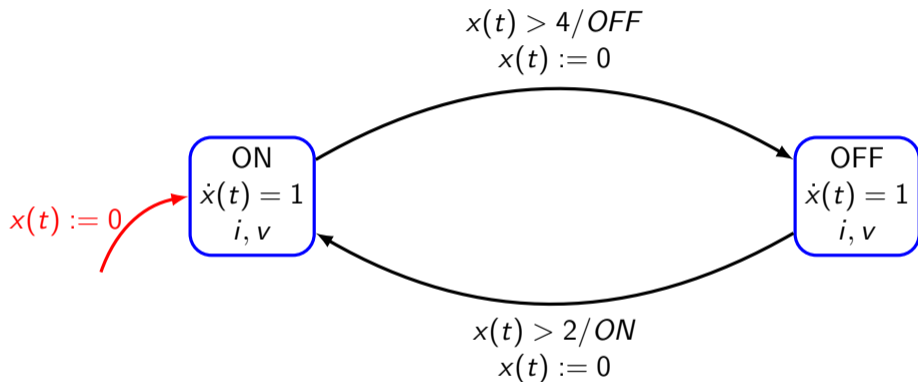
# Buck converter: Mode 1



## Buck converter: Mode 2



# Buck converter: hybrid automata



# Automatic cruise control

right

$\dot{x}, \dot{y}, \dot{\theta}$

$$e = f(x, y)$$

straight

$\dot{x}, \dot{y}, \dot{\theta}$

$$e = f(x, y)$$

stop

$\dot{x}, \dot{y}, \dot{\theta}$

$$e = f(x, y)$$

left

$\dot{x}, \dot{y}, \dot{\theta}$

$$e = f(x, y)$$

# Automatic cruise control

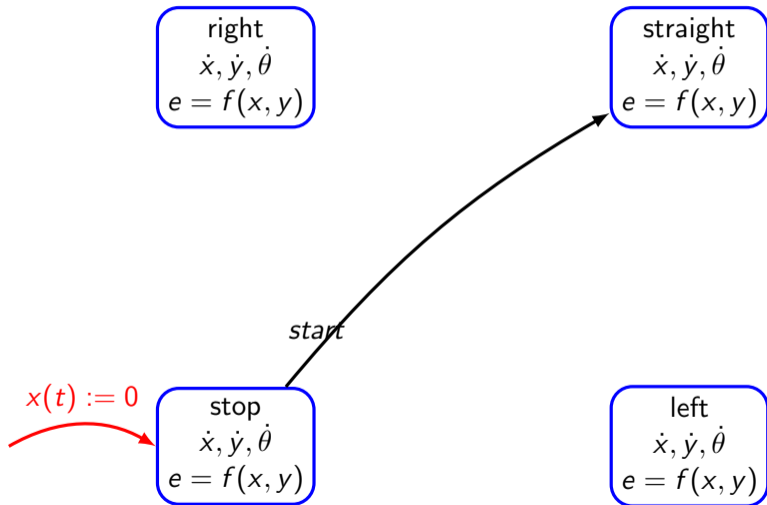
right  
 $\dot{x}, \dot{y}, \dot{\theta}$   
 $e = f(x, y)$

straight  
 $\dot{x}, \dot{y}, \dot{\theta}$   
 $e = f(x, y)$

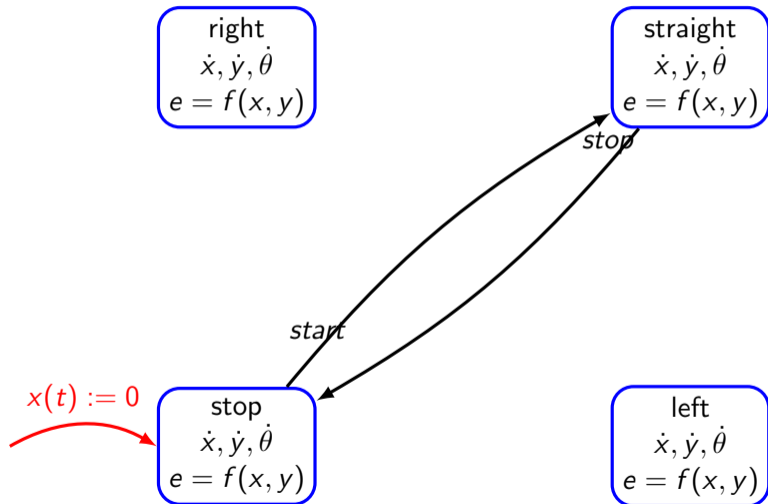
$x(t) := 0$   
stop  
 $\dot{x}, \dot{y}, \dot{\theta}$   
 $e = f(x, y)$

left  
 $\dot{x}, \dot{y}, \dot{\theta}$   
 $e = f(x, y)$

# Automatic cruise control

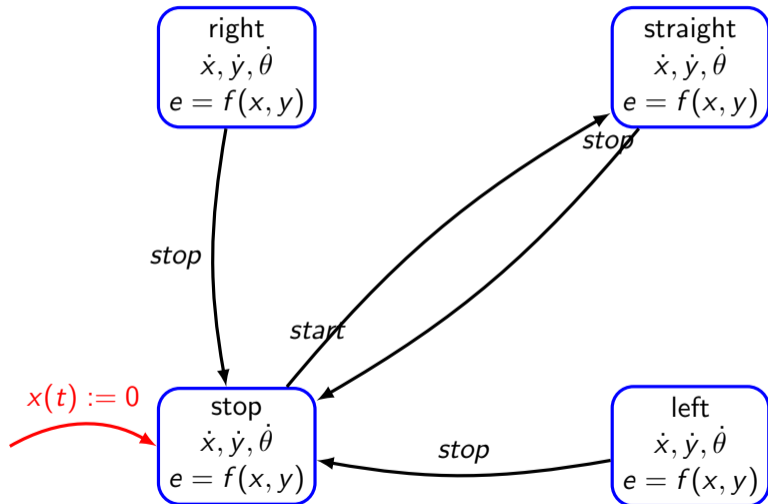


# Automatic cruise control

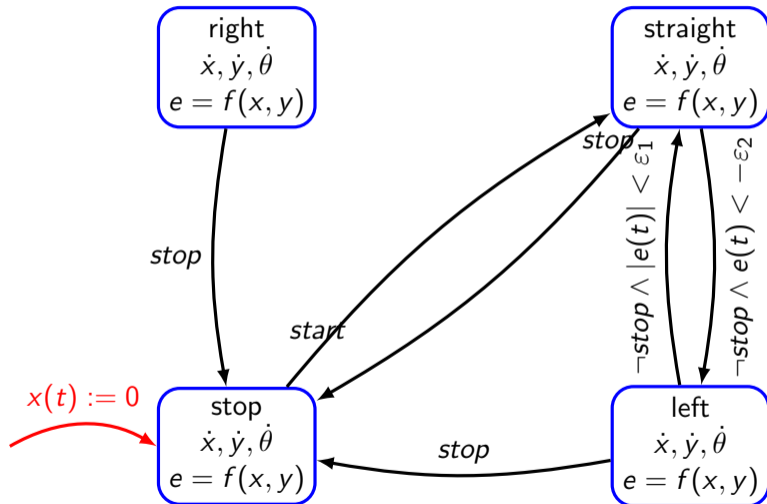




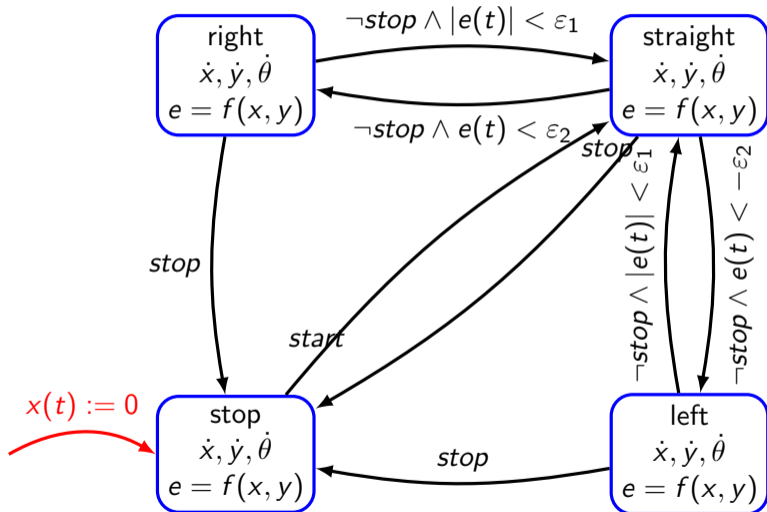
# Automatic cruise control



# Automatic cruise control



# Automatic cruise control



# Automatic cruise control

