# Introduction to Deep Learning

**Arijit Mondal**
**Dept. of Computer Science & Engineering**
**Indian Institute of Technology Patna**
arijit@iitp.ac.in

# Neural Network

# Human brain vs von Neumann computer

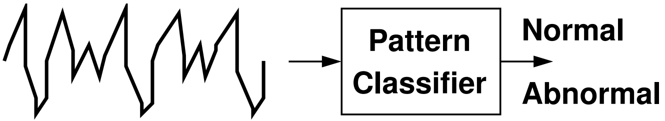
- Massive parallelism
- Distributed representation and computation
- Learning ability
- Generalization ability
- Adaptability
- Inherent contextual information processing
- Fault tolerance
- Low energy consumption

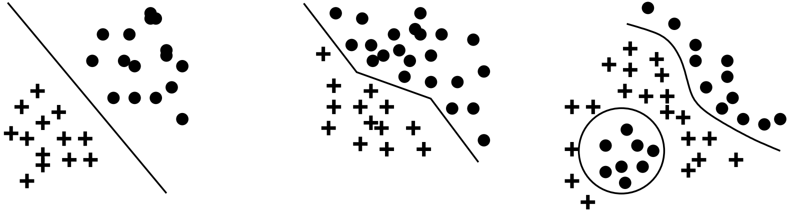
# Computer vs Brain

| | von Neumann | Neural system |
|---|---|---|
| Processor | Complex, high speed, one or a few | Simple, low speed, a large number |
| Memory | Separate from processor, Localized, Noncontent addressable | Integrated into processor, Distributed, Content addressable |
| Computing | Centralized, sequential, stored program | Distributed, parallel, self-learning |
| Reliability | Very vulnerable | Robust |
| Expertise | Numeric and symbolic manipulations | Perceptual problems |
| Operating environment | Well defined, well constrained | Poorly defined, unconstrained |

# Artificial Neuron: Applications

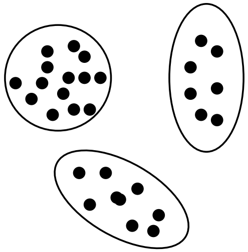
- **Pattern classification**



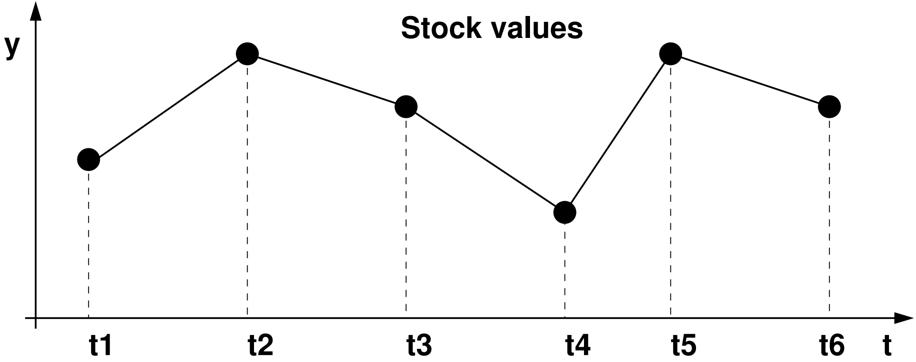
# Artificial Neuron: Applications

- Clustering/categorization



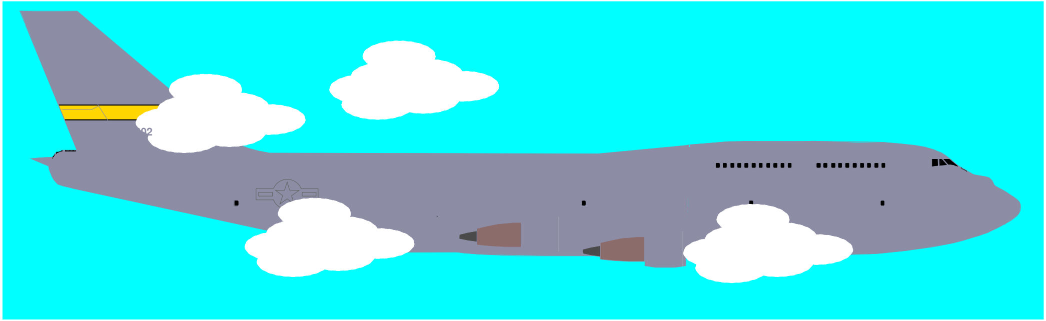
# Artificial Neuron: Applications

- **Prediction**



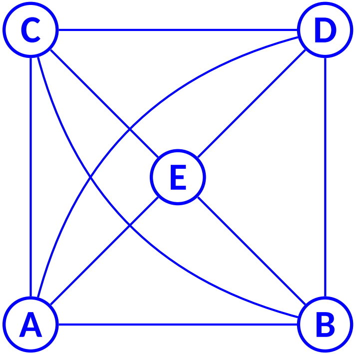
# Artificial Neuron: Applications

- Retrieval



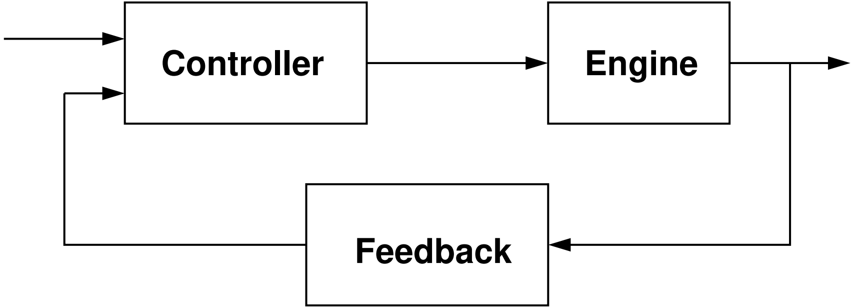
# Artificial Neuron: Applications

- Optimization



# Artificial Neuron

- Control



# History

- Started in 1940s by McCulloch and Pitt
- Rosenblatt perceptron convergence theorem (1960)
- In 1980s ANN started gaining popularity
- Again became popular after 2006

# Biological Neuron

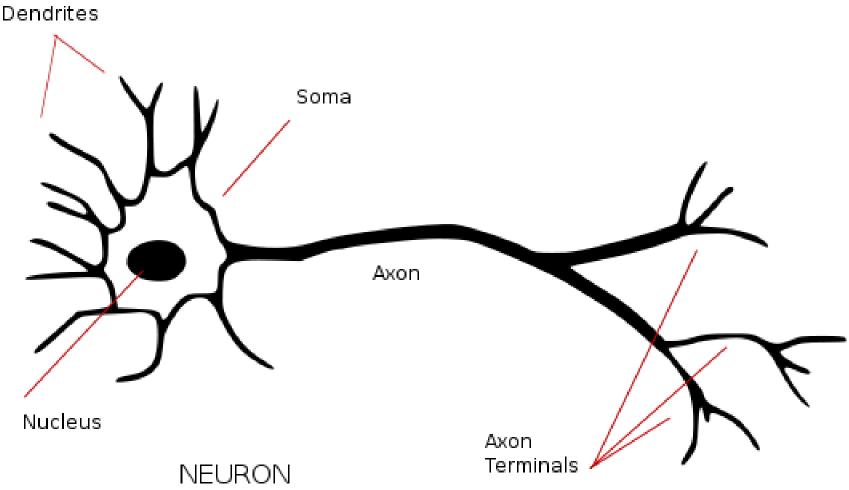



Image source: Internet

# Cerebral cortex

- It is a flat sheet of neurons about 2-3 millimeter thick with surface area is 2200 $cm^2$
  - Twice the area of computer keyboard
- It contains around $10^{11}$ neurons
  - Number of stars in the Milky-way
- Each neuron is connected to $10^3$-$10^4$ other neurons
- Total connections is around $10^{14}$-$10^{15}$
- Connectionist model

# Human brain

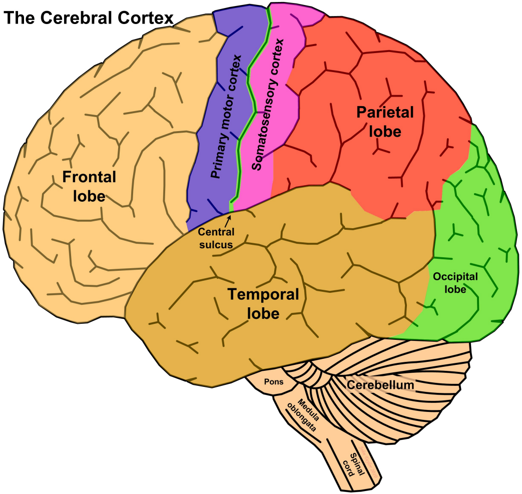
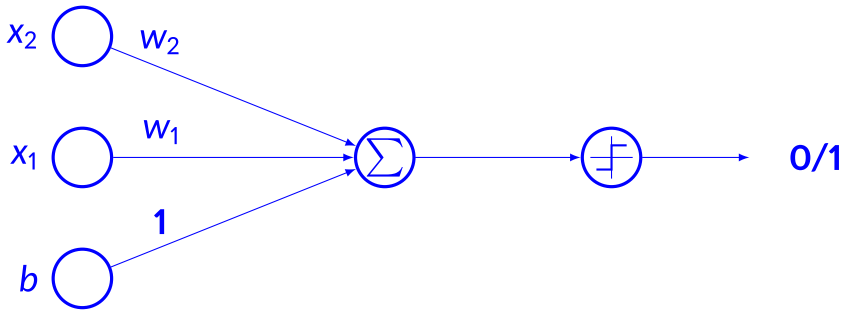



Image source: Internet

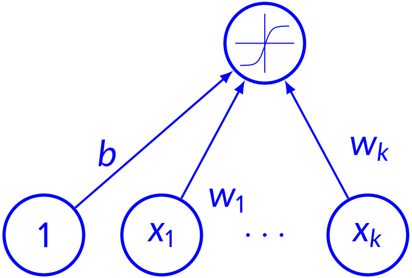
# Neuron

- One of the primitive models

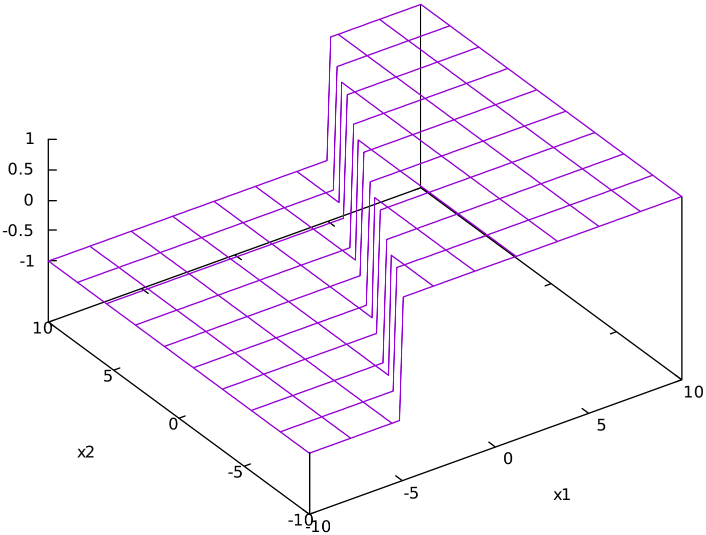


# Artificial Neuron

- **Neuron pre-activation function**
  - $a(\mathbf{x}) = \sum_i w_i x_i + b = b + \mathbf{w}^T \mathbf{x}$
- **Neuron output activation function**
  - $h(\mathbf{x}) = g(a(\mathbf{x})) = g\left(\sum_i w_i x_i + b\right)$
- **Notations**
  - $\mathbf{w}$ — Weight vector
  - $b$ — Neuron bias
  - $g(.)$ — Activation function

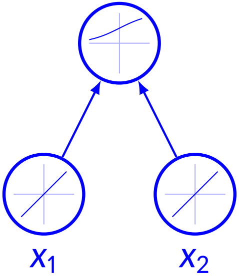


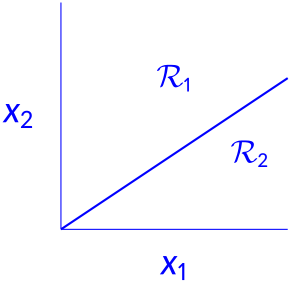
# Physical interpretation



# Classification using single neuron

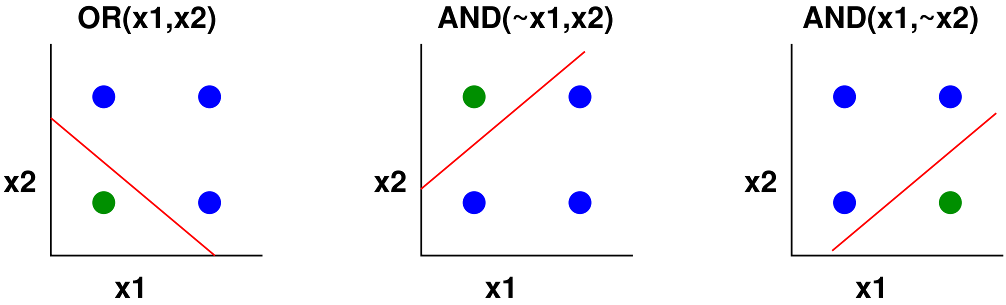
- Single neuron can do binary classification
  - Also known as logistic regression classifier



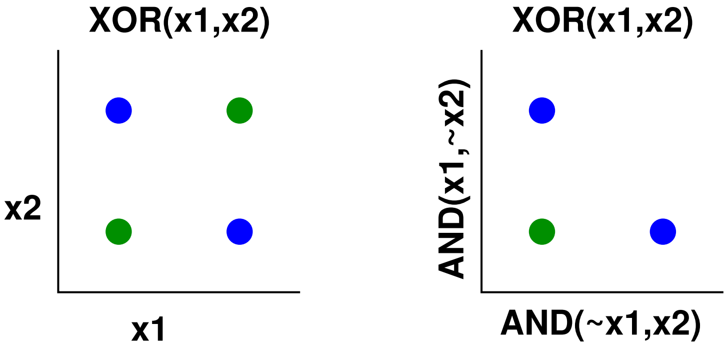
# Artificial neuron

- Can solve linearly separable problems



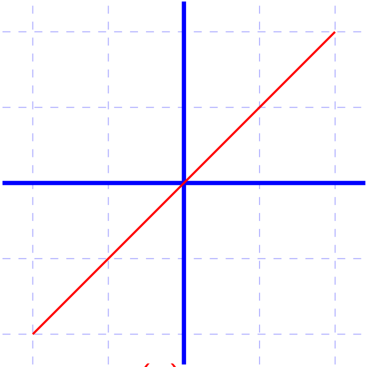
# Artificial neuron: XOR problem

- There are issues for linear separation



# Activation function

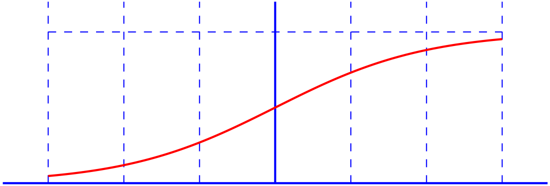
- **Linear activation function**
  - Not very interesting
  - No change in values
  - Huge range



$g(x) = x$

# Activation function

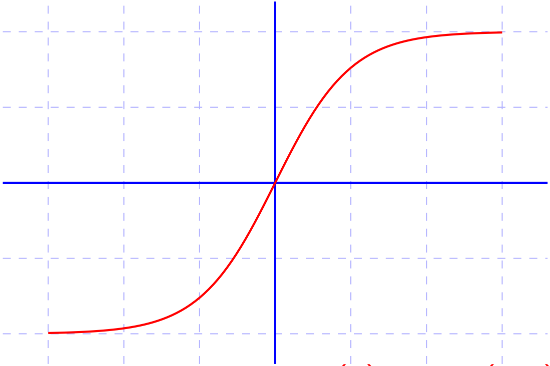
- **Sigmoid function**
  - Values lie between 0 and 1
  - Strictly increasing function
  - Bounded



$$g(x) = \mathbf{sigm}(x) = \frac{1}{1 + \exp(-x)}$$

# Activation function

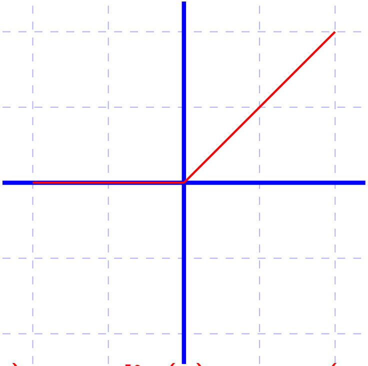
- **Hyperbolic Tangent (Tanh) function**
  - Can be positive or negative
  - Values lie between -1 and 1
  - Strictly increasing function
  - Bounded



$$g(x) = \mathbf{tanh}(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

# Activation function

- Rectified linear activation function
  - Bounded below by 0
  - Strictly increasing function
  - Not upper bounded



$$g(x) = \mathbf{reclin}(x) = \max(0, x)$$

# Single hidden layer neural network

- **Hidden layer pre-activation**

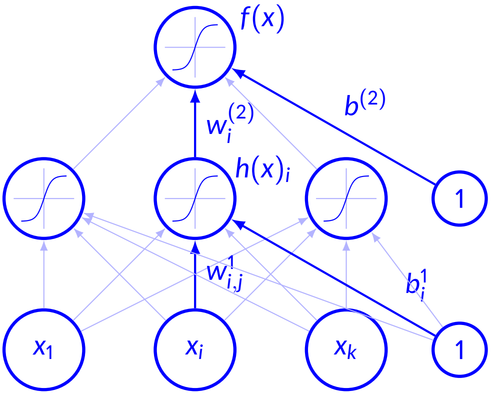
$$\boldsymbol{a}(\boldsymbol{x}) = \boldsymbol{b}^1 + \boldsymbol{w}^1\boldsymbol{x}$$

- **Hidden layer activation**

$$h(\boldsymbol{x}) = g(a(\boldsymbol{x}))$$

- **Output layer activation**

$$f(\boldsymbol{x}) = o(b^{(2)} + \boldsymbol{w}^{(2)T}h^1(\boldsymbol{x}))$$



# Multi layer neural network

- **Pre-activation in layer**

  $k > 0 \ (\boldsymbol{h}^{(0)}(\boldsymbol{x}) = x)$

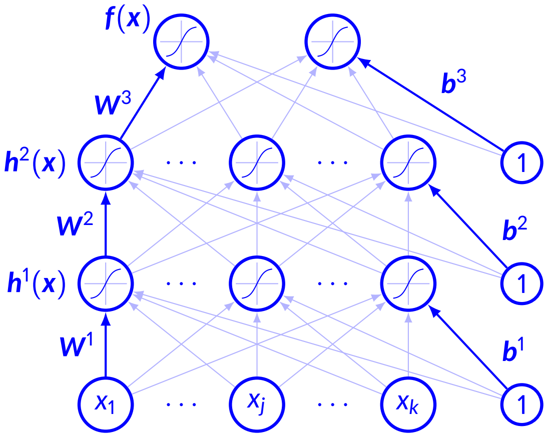
  $$\boldsymbol{a}^{(k)}(\boldsymbol{x}) = \boldsymbol{b}^{(k)} + \boldsymbol{W}^{(k)}\boldsymbol{h}^{(k-1)}\boldsymbol{x}$$

- **Hidden layer activation**

  $$\boldsymbol{h}^{(k)}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{a}^{(k)}(\boldsymbol{x}))$$

- **Output layer activation**

  $$\boldsymbol{h}^{(L+1)}(\boldsymbol{x}) = o(\boldsymbol{a}^{(L+1)}(\boldsymbol{x})) = \boldsymbol{f}(\boldsymbol{x})$$



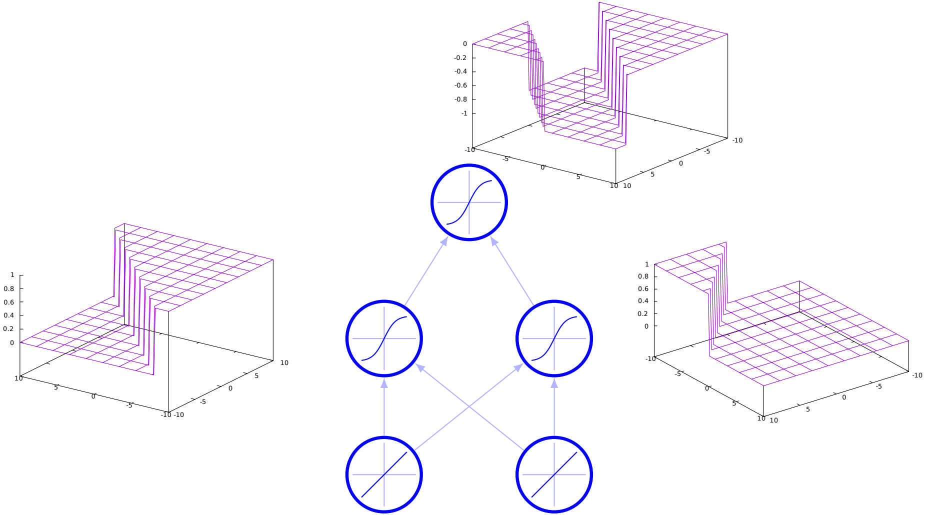
# Multiclass classification

- Need multiple outputs that is one neuron for each class
- Need to determine probability of $p(y = c|\boldsymbol{x})$
- Softmax activation function is used at the output

$$\boldsymbol{o}(\boldsymbol{a}) = \textbf{softmax}(a) = \left[ \frac{\exp(a_1)}{\sum_c \exp(a_c)} \quad \frac{\exp(a_2)}{\sum_c \exp(a_c)} \quad \dots \quad \frac{\exp(a_c)}{\sum_c \exp(a_c)} \right]^T$$

- Strictly positive
- Sum to 1
- Class having the highest probability will be the predicted output

# Capacity of neural network



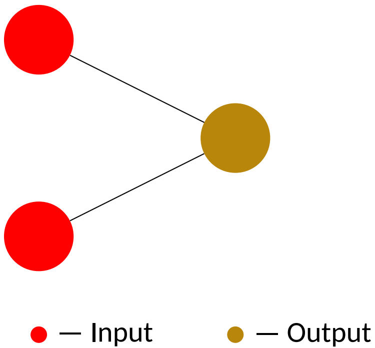
# Capacity of neural network

- Universal approximation theorem (Hornik,1991)
  - A single hidden layer neural network with a linear output unit can approximate any continuous function arbitrarily well, given enough hidden units.
- The result is applicable for other hidden layer activation functions such as sigmoid, tanh, etc.
- This is a promising result, but it does not say that there is a learning algorithm to find the necessary parameter values!

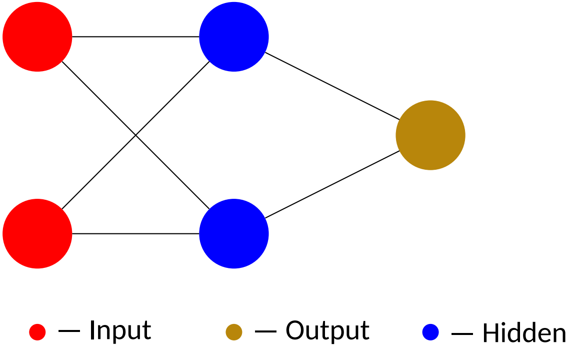
# Types of Neural Network

- Feed forward neural network
- Radial basis function network
- Recurrent neural network
- Boltzmann machine
- Long short term memory network
- and many more

# Perceptron

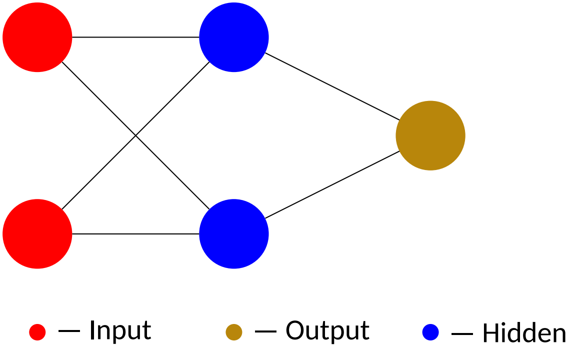


# Feed Forward

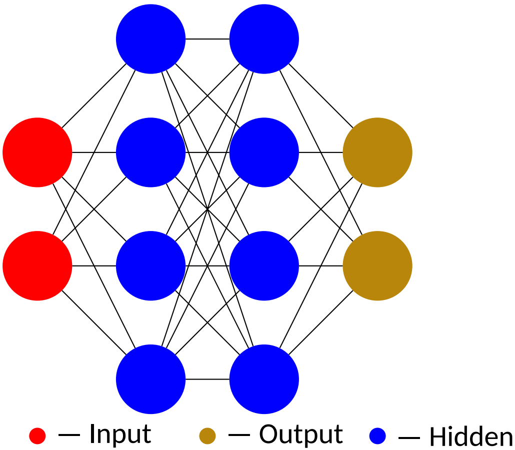


# Radial Basis Function

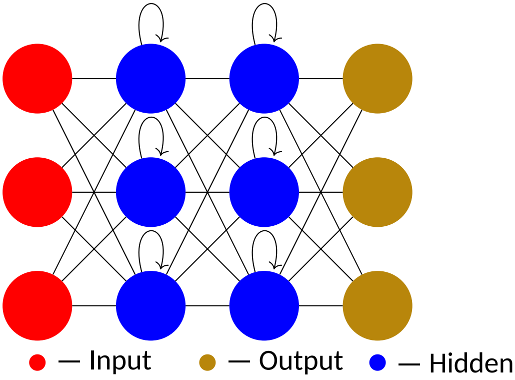
- **Typically it will have 3 layers**
- **Distance from a center vector is computed**



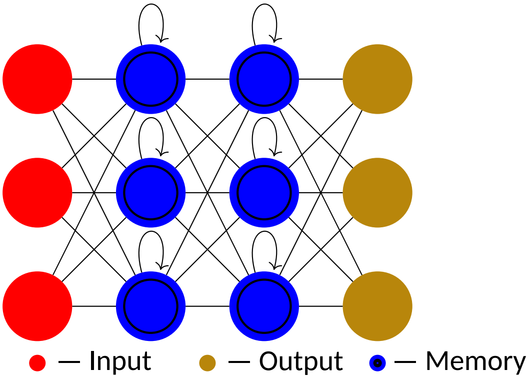
# Deep Feed Forward



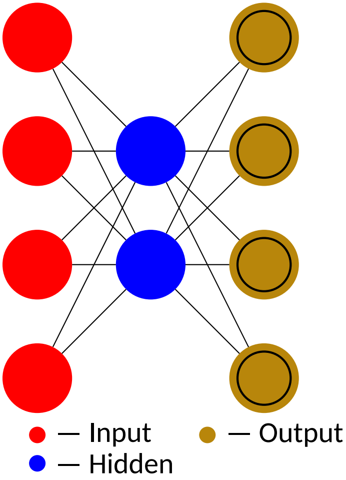
# Recurrent Neural Network



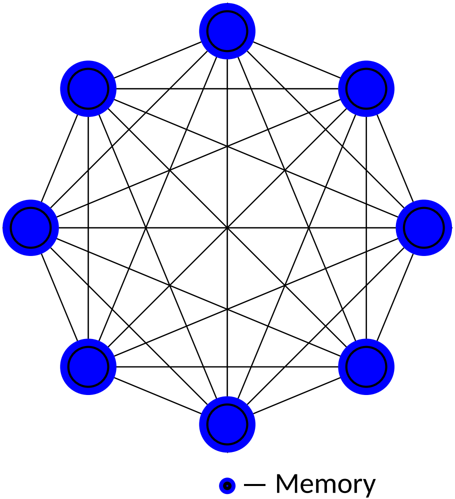
# Long Short Term Memory



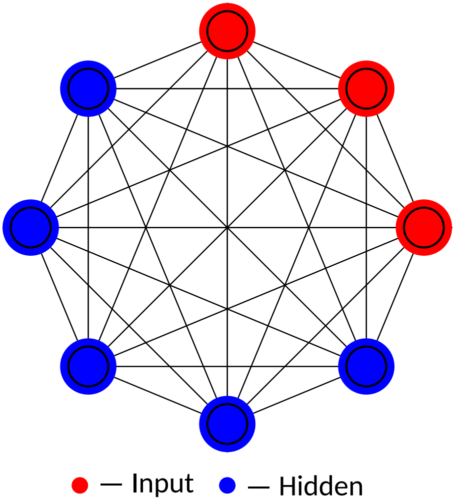
# Auto Encoder



# Markov chain



# Boltzmann Machine



# Learning the parameters

- The network must learn the connection weights from available training examples
- Learning can be
  - Supervised
  - Unsupervised
  - Hybrid
- Four basic types of learning rule
  - Error correction rule
  - Boltzmann learning
  - Hebbian
  - Competitive learning

# Error correction rule

- Output is generated based on the weight values but this may vary from desired value
- The error information is used to update the weight value
- Perceptron learning algorithm
  - Initialize the weights and threshold to small random numbers
  - Present a pattern vector and evaluate the output of neuron
  - Update the weight according to $w_j(t+1) = w_j(t) + \eta(d - y)x_j$
- Back propagation algorithm

# Boltzmann learning

- Usually symmetric recurrent network consisting of binary units
- A subset of neurons interact with environment
- Generally it has two modes
  - Clamped — Visible neurons are clamped to specific states
  - Free-running - Visible and hidden unit operate freely
- Stochastic learning rule derived from information theoretic and thermodynamic principles
- Learning rule is given by $\Delta w_{ij} = \eta(\bar{\rho}_{ij} - \rho_{ij})$

# Hebbian rule

- One of the oldest learning rules
- If neuron on both sides of a synapse are activated synchronously and repeatedly, the synapse's strength is selectively increased
- Mathematically, it can be described as $w_{ij}(t+1) = w_{ij}(t) + \eta y_j(t) x_i(t)$

# Competitive learning rule

- Output units compete among themselves for activation
- Only one output is active at time
- Also known as winner-take-all
- Mathematically, it can be represented as $\boldsymbol{w}_{i^*}\boldsymbol{x} \geq \boldsymbol{w}_i\boldsymbol{x}$
- Competitive learning rule can be stated as

$$\Delta w_{ij} = \begin{cases} \eta(x_j^u - w_{i^*j}) & i = i^* \\ 0 & i \neq i^* \end{cases}$$

# Summary

- Error correction rule — Single or multilayer perceptron
  - Pattern classification, function approximation, prediction, control
- Boltzmann — Recurrent
  - Pattern classification
- Hebbian — Multilayer feed forward
  - Pattern classification, data analysis
- Competitive
  - Within class categorization, data compression