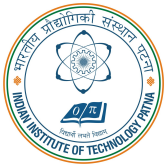
# Quantitative analysis

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# Introduction

- Quantitative properties
  - Position, velocity, temperature, weight, reaction time, etc.
- Execution time analysis
- Power analysis
- Need to analyze program / software
- Require models of platform

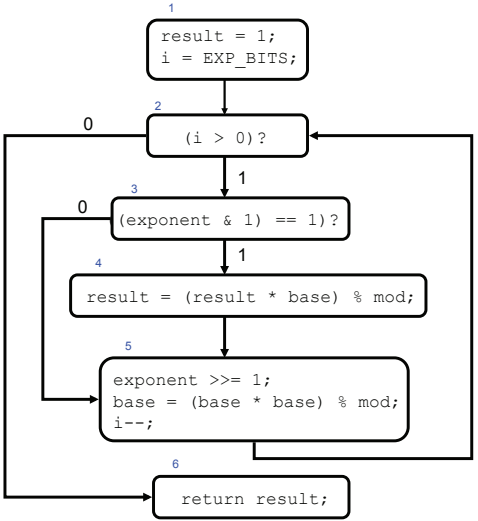
# Target

- Generic problem
  - A task defined by a program $P$, environment $E$ and quantity of interest $q$, then $q = f_p(x, w)$
    - $x$ - input to program
    - $w$ - environment parameters
- In most of the cases extreme value of the quantity is of interest
- Extreme case analysis: $\max_{x,w} f_p(x, w)$ or $\min_{x,w} f_p(x, w)$
- It may be difficult to determine exact value
- Threshold analysis: $\forall x, w \ f_p(x, w) \leq T$ or $\forall x, w \ f_p(x, w) \geq T$
- Average case analysis: $E_{D_x, D_w} f_p(x, w)$

# Modular exponentiation: $b^e \bmod n$

```
#define EXP_BITS 32
typedef unsigned int UI;
UI modexp(UI base, UI exponent, UI mod) {
  int i;
  UI result = 1;
  i = EXP_BITS;
  while(i > 0) {
    if ((exponent & 1) == 1) {
      result = (result * base) % mod;
    }
    exponent >>= 1;
    base = (base * base) % mod;
    i--;
  }
  return result;
}
```

# Control flow diagram

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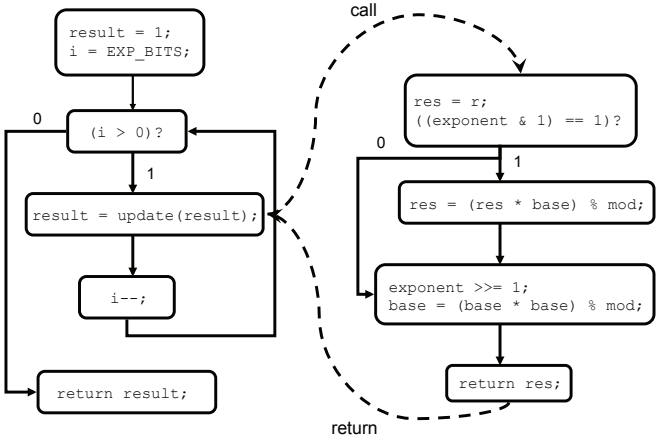
# Modular exponentiation: $b^e \bmod n$

```
#define EXP_BITS 32
typedef unsigned int UI;
UI exponent, base, mod;
UI update(UI r) {
  UI res = r;
  if ((exponent & 1) == 1) { res = (res * base) % mod; }
  exponent >>= 1;
  base = (base * base) % mod;
  return res;
}
UI modexp_call() {
  UI result = 1; int i;
  i = EXP_BITS;
  while(i > 0) { result = update(result); i--; }
  return result;
}
```

# Control flow diagram

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# Execution time: Loop bounds

```
for(i=EXP_BITS; i > 0; i--) {
  if ((exponent & 1) == 1) {
    result = (result * base) % mod;
  }
  exponent >>= 1;
  base = (base * base) % mod;
}

while (exponent != 0) {
  if ((exponent & 1) == 1) {
    result = (result * base) % mod;
  }
  exponent >>= 1;
  base = (base * base) % mod;
}
```

# Execution time: Exponential path space

```
void count() {
  int Outer, Inner;
  for (Outer = 0; Outer < MAXSIZE; Outer++) {
    for (Inner = 0; Inner < MAXSIZE; Inner++){
      if (Array[Outer][Inner] >= 0) {
        Ptotal += Array[Outer][Inner];
        Pcnt++;
      } else {
        Ntotal += Array[Outer][Inner];
        Ncnt++;
      }
    }
  }
}
```

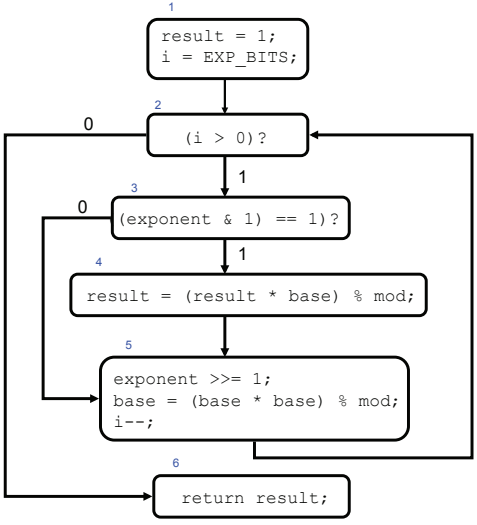
# Execution time: Path feasibility

```
void altitude_control_task(void) {
  if (pprz_mode == PPRZ_MODE_AUTO2 || pprz_mode == PPRZ_MODE_HOME) {
    if (vertical_mode == VERTICAL_MODE_AUTO_ALT) {
      float err = estimator_z - desired_altitude;
      desired_climb = pre_climb + altitude_pgain * err;
      if (desired_climb < -CLIMB_MAX) {
        desired_climb = -CLIMB_MAX;
      }
      if (desired_climb > CLIMB_MAX) {
        desired_climb = CLIMB_MAX;
      }
    }
  }
}
```

# Execution time: Memory hierarchy

```
float dot_product(float *x, float *y, int n) {
  float result = 0.0;
  int i;
  for(i=0; i < n; i++) {
    result += x[i] * y[i];
  }
  return result;
}
```

# Constraints for CFG




$$
\begin{aligned}
x_1 &= 1 \\
x_6 &= 1 \\
x_1 &= d_{12} \\
x_2 &= d_{12} + d_{52} = d_{23} + d_{26} \\
x_3 &= d_{23} = d_{34} + d_{35} \\
x_4 &= d_{34} = d_{45} \\
x_5 &= d_{35} + d_{45} = d_{52} \\
x_6 &= d_{26}
\end{aligned}
$$

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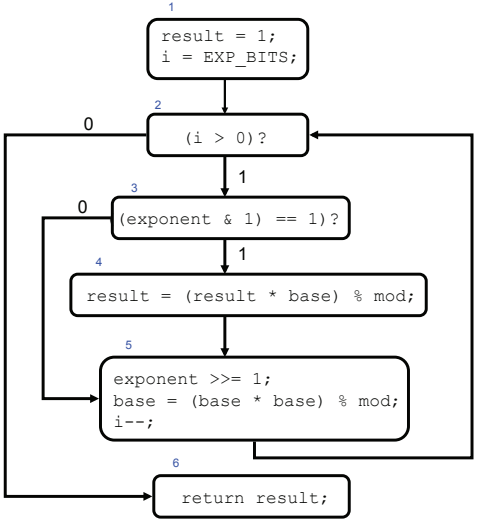
# Constraints for CFG




$$x_1 = 1$$
$$x_6 = 1$$
$$x_1 = d_{12}$$
$$x_2 = d_{12} + d_{52} = d_{23} + d_{26}$$
$$x_3 = d_{23} = d_{34} + d_{35}$$
$$x_4 = d_{34} = d_{45}$$
$$x_5 = d_{35} + d_{45} = d_{52}$$
$$x_6 = d_{26}$$

$$\max_{x_i, 1 \leq i \leq n} \sum_i w_i x_i$$

Image source: Embedded Systems book