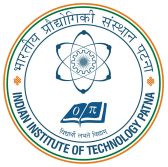
Composition of State Machine



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Introduction

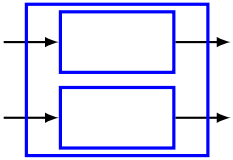
- A system can have large number of states
- Dividing large state machine into smaller can ease modeling
- Multiple state machines interact with each other to realize bigger functionality
- Known as **concurrent composition**
- Hybrid systems are example of **sequential composition**

Concurrent composition

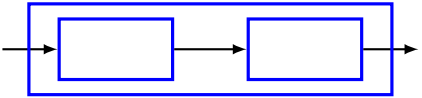
- Two or more state machines react either simultaneously or independently
- **Synchronous composition** — if the reaction occurs simultaneously
- **Asynchronous composition** — if the reaction occurs independently
- Need to define semantics of such composition

Spatial composition

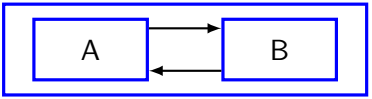
- Side-by-side composition
- Cascade composition
- Feedback composition



side-by-side

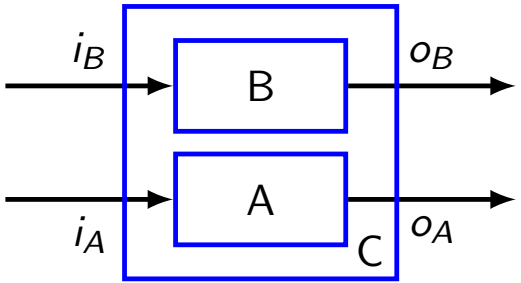


cascade



feedback

Side-by-side synchronous composition



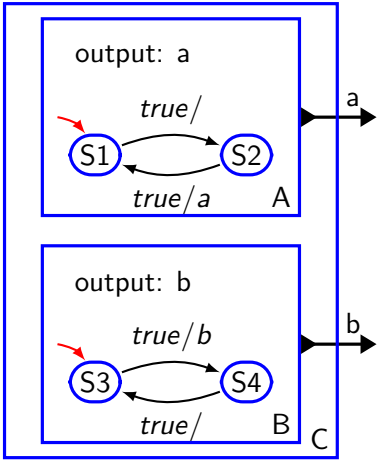
Synchronous composition

- A reaction in the composition system is a simultaneous reaction of its constituents
- Recall, environment decides when state machine reacts
- Synchronous side-by-side composition is easy
- Such compositions are **modular**
- If A and B are deterministic, then so is synchronous side-by-side composition
- **Compositional** — if a property held by the components is also a property of the composition
  - Determinism is compositional

Mathematical description

- Let $SM_A = \langle S_A, I_A, O_A, update_A, S_A^0 \rangle$
- Let $SM_B = \langle S_B, I_B, O_B, update_B, S_B^0 \rangle$
  - S - states, I - inputs, O - outputs, S^0 - initial state
- Composition machine C will have
  - $S_C = S_A \times S_B$
  - $I_C = I_A \times I_B$
  - $O_C = O_A \times O_B$
  - $S_C^0 = S_A^0 \times S_B^0$
  - $update_C((s_A, s_B), (i_A, i_B)) = ((s'_A, s'_B), (o_A, o_B))$ where $(s'_A, o_A) = update_A(s_A, i_A)$ and $(s'_B, o_B) = update_B(s_B, i_B)$

Side-by-side synchronous composition: example



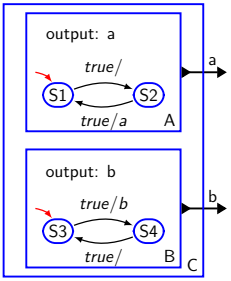
Side-by-side synchronous composition: example

S1,S3

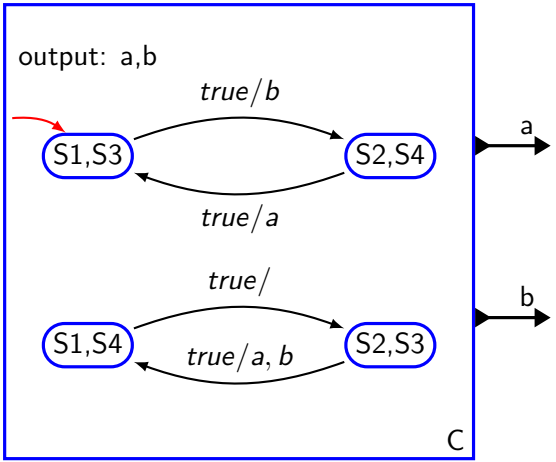
S2,S4

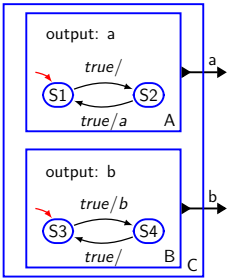
S1,S4

S2,S3



Side-by-side synchronous composition: example





Side-by-side asynchronous composition

- Component machine reacts independently
- A reaction of composition machine C is a reaction of one of A or B where the choice is nondeterministic
  - Interleaving in nature as A or B never react simultaneously
- A reaction of composition machine C is a reaction of A or B or both A and B where the choice is nondeterministic
- Composition machine C will have
  - $update_C((s_A, s_B), (i_A, i_B)) = ((s'_A, s'_B), (o'_A, o'_B))$ where
    $(s'_A, o'_A) = update_A(s_A, i_A)$ and $s'_B = s_B$ and $o'_B = absent$
    OR
    $(s'_B, o'_B) = update_B(s_B, i_B)$ and $s'_A = s_A$ and $o'_A = absent$

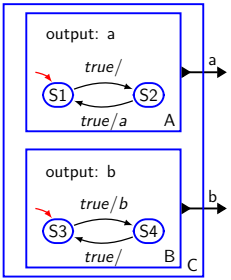
Side-by-side asynchronous composition:example

S1,S3

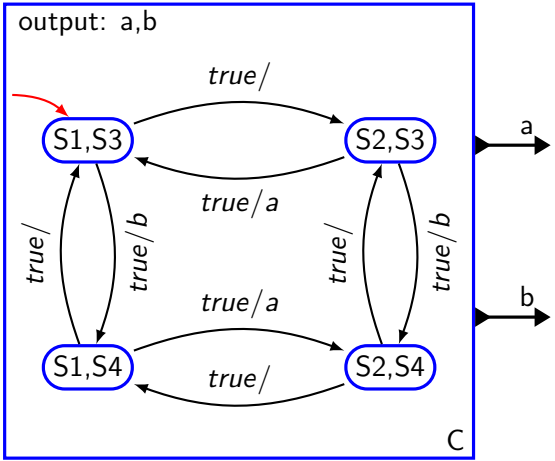
S2,S3

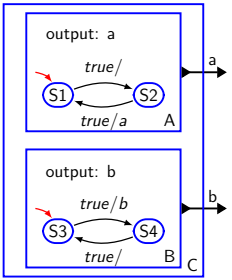
S1,S4

S2,S4



Side-by-side asynchronous composition:example





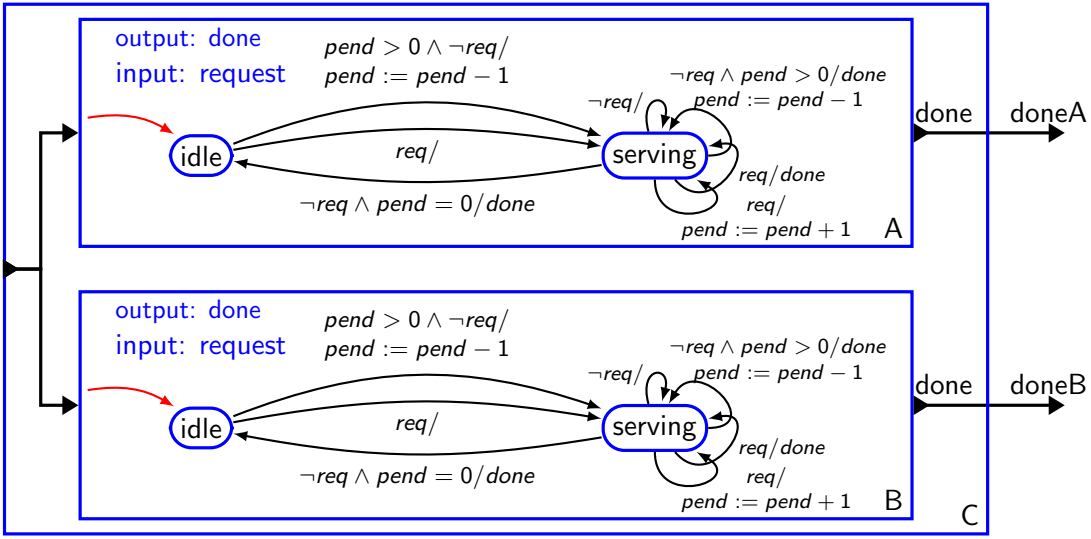
Composition using shared variable

- Extended state machine has local variables
- Sometime it is useful when composing state machine to allow these variables shared
  - Useful for modeling interrupts, threads, etc.
- Atomic operation

Composition using shared variable: example

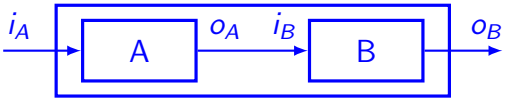
- Two servers receive request
- Each request requires unknown amount of time of service
- Server shares queue of request
- If one server is busy, the other server can respond yo a request, even if the request arrives at the network interface of the first server

Server queue



Cascade composition

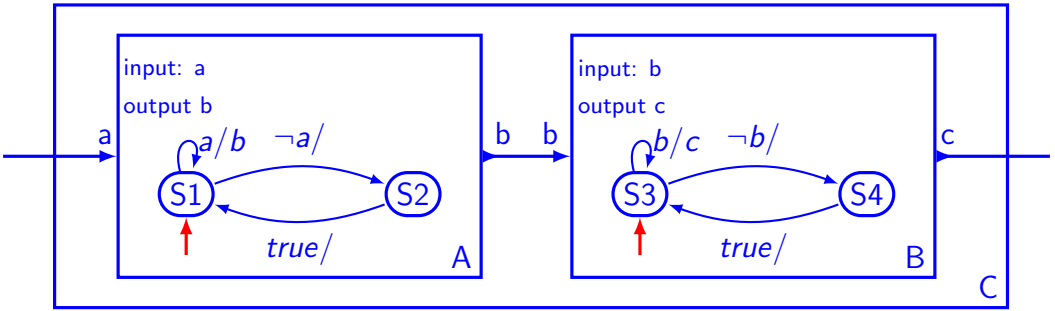
- Output of one of the machines feeds into input of the other



- Data type must conform
- Synchronous composition

Cascade composition:Example

- Output of one of the machines feeds into input of the other



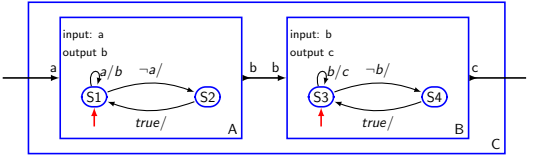
Cascade composition:synchronous

S1,S3

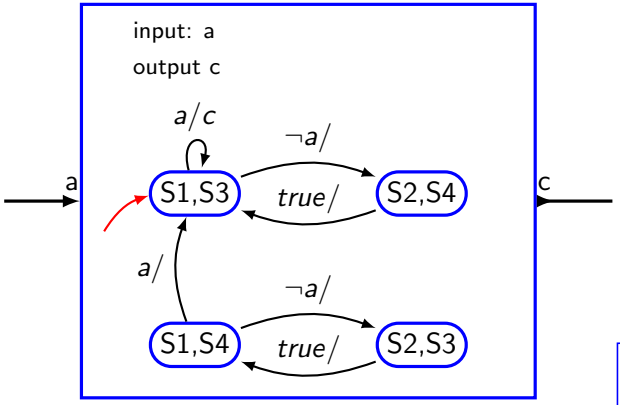
S2,S4

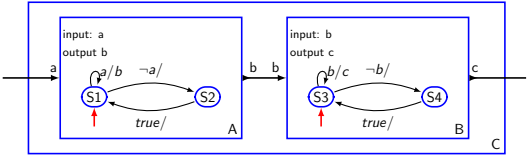
S1,S4

S2,S3

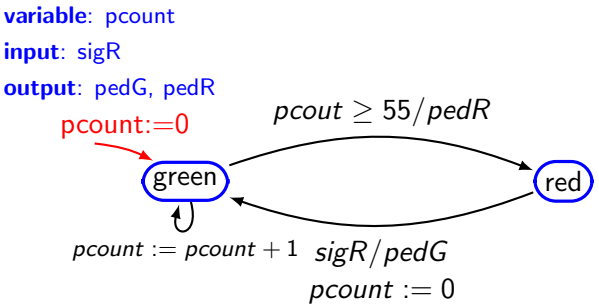


Cascade composition:synchronous

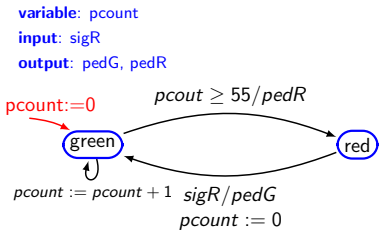


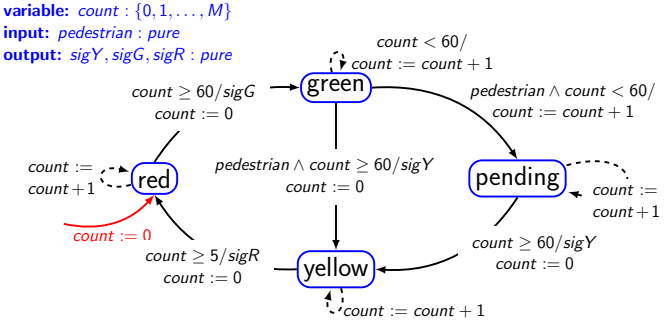


Cascade composition:Traffic light



Cascade composition:Traffic light



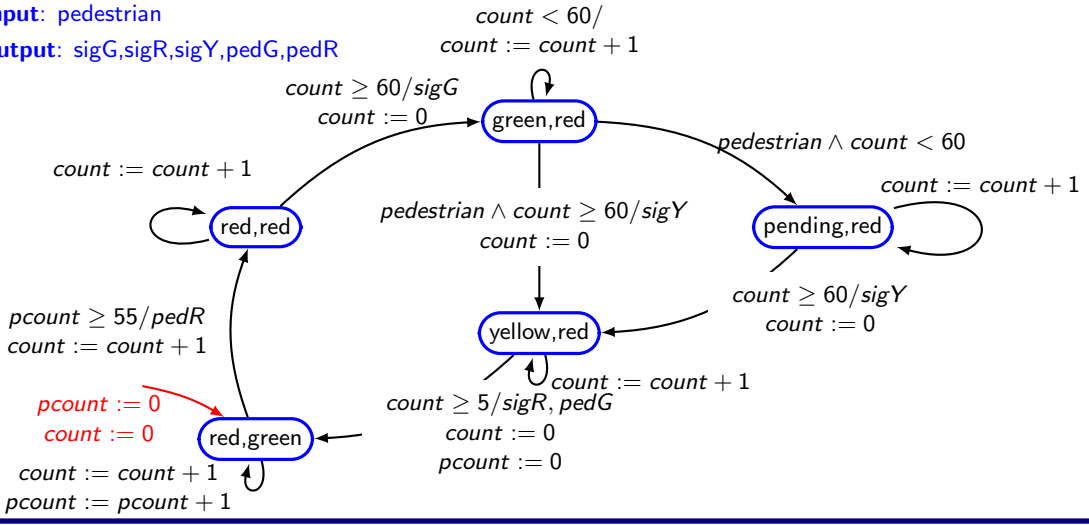


Cascade composition:Traffic light

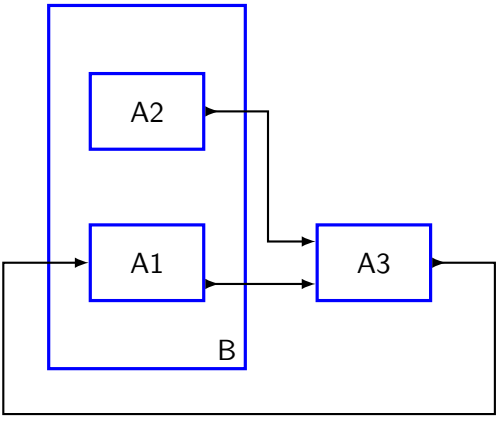
variable: pcount,count

input: pedestrian

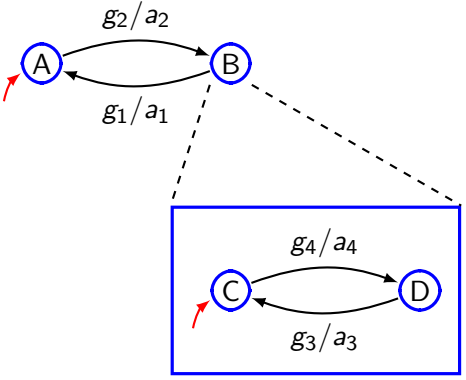
output: sigG,sigR,sigY,pedG,pedR



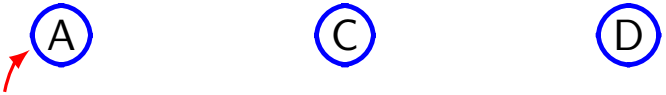
General composition

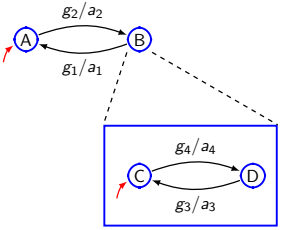


Hierarchical SM

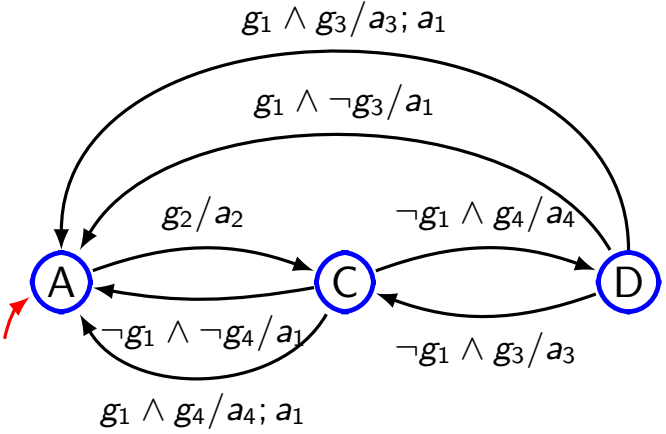


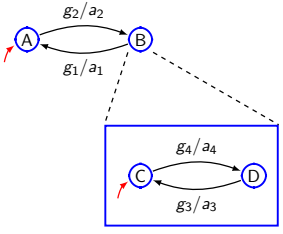
Hierarchical SM



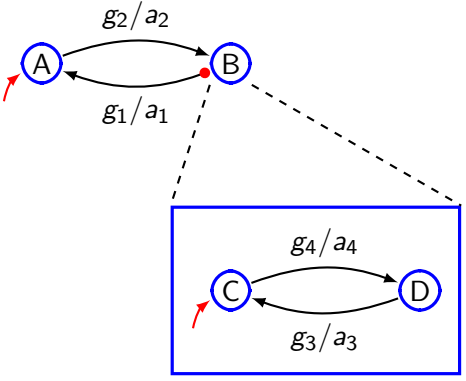


Hierarchical SM

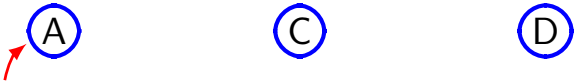


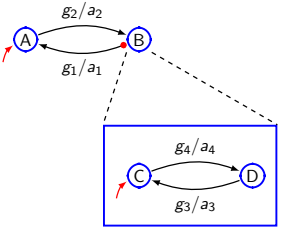


Hierarchical SM: preemptive transition

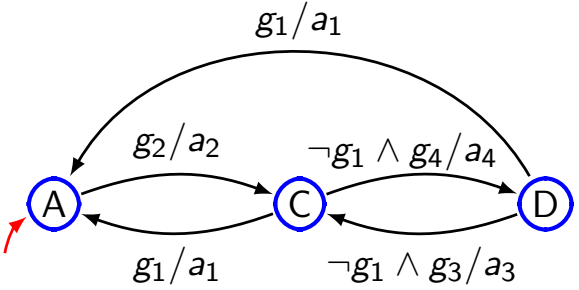


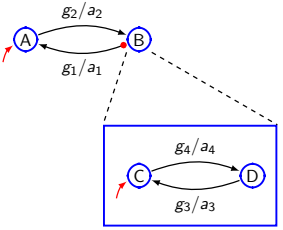
Hierarchical SM: preemptive transition



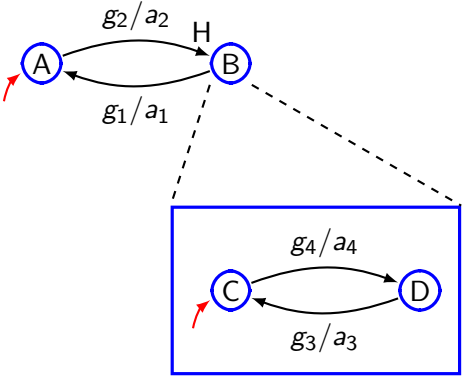


Hierarchical SM: preemptive transition

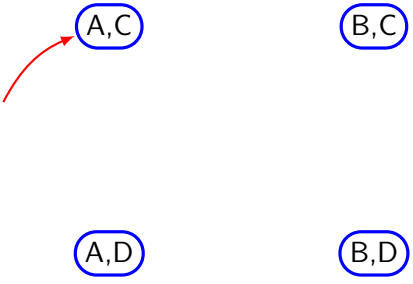


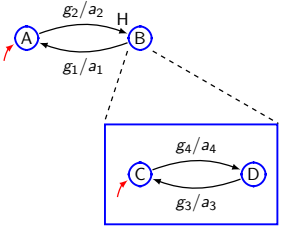


Hierarchical SM: history transition



Hierarchical SM: history transition





Hierarchical SM: history transition