# Introduction to Deep Learning

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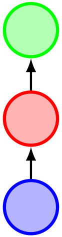
# Recurrent Neural Network

# Introduction

- Recurrent neural networks are used for processing sequential data in general
  - Convolution neural network is specialized for image
- Capable of processing variable length input
- Shares parameters across different part of the model
  - Example: "I went to IIT in 2017" or "In 2017, I went to IIT"
  - For traditional machine learning models require to learn rules for different positions

# Types of applications



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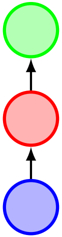

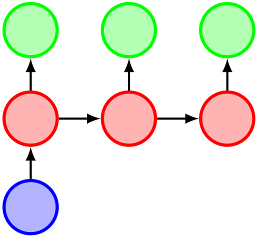

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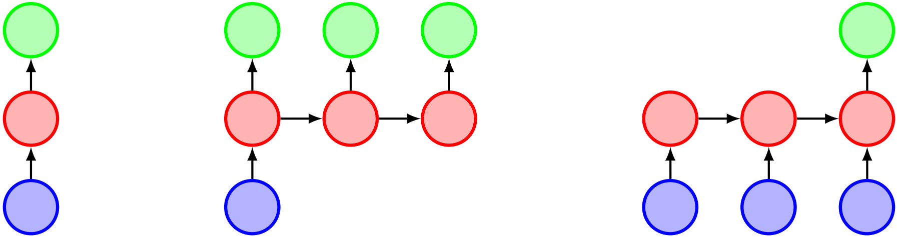

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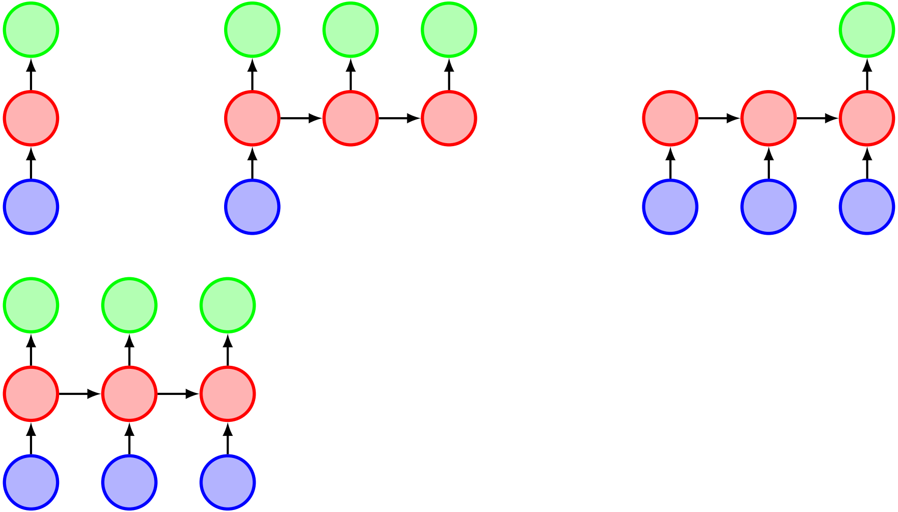

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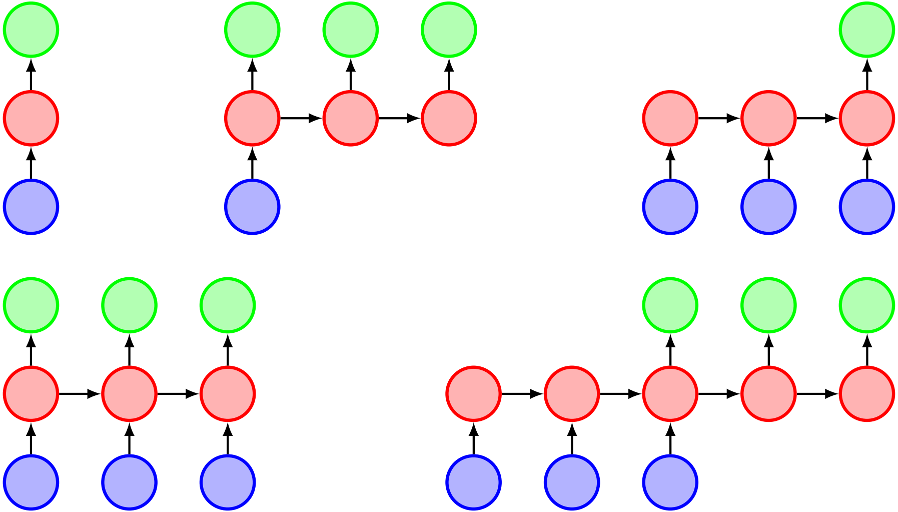
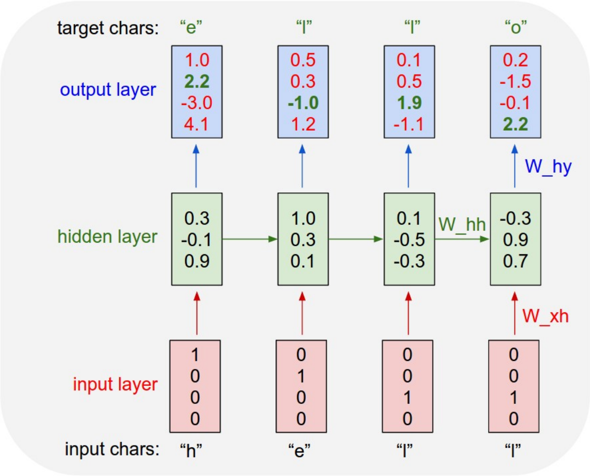


# Example



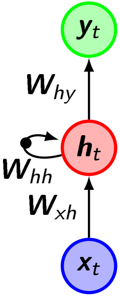
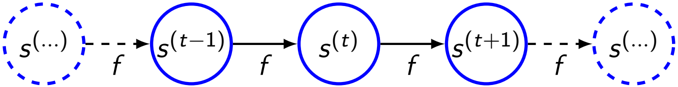




Image source: http://karpathy.github.io/2015/05/21/rnn-effectiveness/

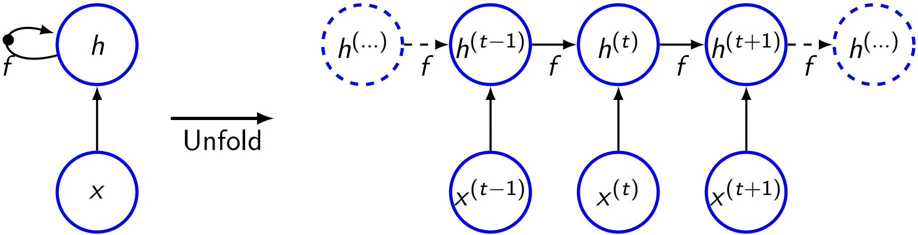
# Computational graph

- Formal way to represent the computation
- Unfolding the graph results in sharing of parameters
- Consider a system $\boldsymbol{s}^{(t)} = f(\boldsymbol{s}^{(t-1)}, \boldsymbol{\theta})$ where $\boldsymbol{s}^{(t)}$ denotes the state of the system
  - It is recurrent
  - For finite number of steps, it can be unfolded
  - Example: $s^{(3)} = f(s^{(2)}, \theta) = f(f(s^{(1)}, \theta), \theta)$



# System with inputs

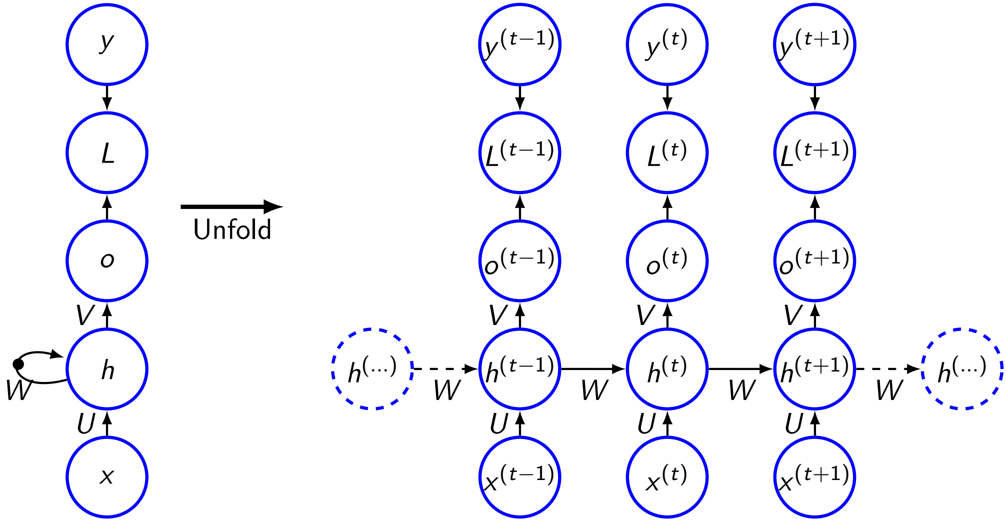
- A system will be represented as $\boldsymbol{s}^{(t)} = f(\boldsymbol{s}^{(t-1)}, \boldsymbol{x}^{(t)}, \boldsymbol{\theta})$
  - A state contains information of whole past sequence
- Usually state is indicated as hidden units such that $\boldsymbol{h}^{(t)} = f(\boldsymbol{h}^{(t-1)}, \boldsymbol{x}^{(t)}, \boldsymbol{\theta})$
- While predicting, network learn $\boldsymbol{h}^{(t)}$ as a kind of lossy summary of past sequence upto $t$
  - $\boldsymbol{h}^{(t)}$ depends on $(\boldsymbol{x}^{(t)}, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})$



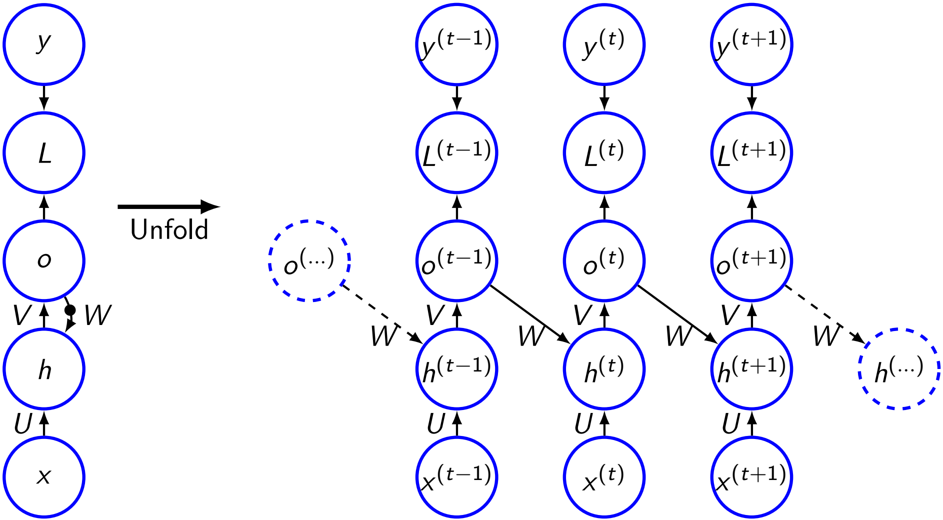
# System with inputs (contd.)

- Unfolded recursion after $t$ steps will be $\boldsymbol{h}^{(t)} = \boldsymbol{g}^{(t)}(\boldsymbol{x}^{(t)}, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)}) = f(\boldsymbol{h}^{(t-1)}, \boldsymbol{x}^{(t)}, \boldsymbol{\theta})$
- Unfolding process has some advantages
  - Regardless of sequence length, learned model has same input size
  - Uses the same transition function $f$ with the same parameters at every time steps
- Can be trained with fewer examples
- Recurrent graph is succinct
- Unfolded graph illustrates the information flow

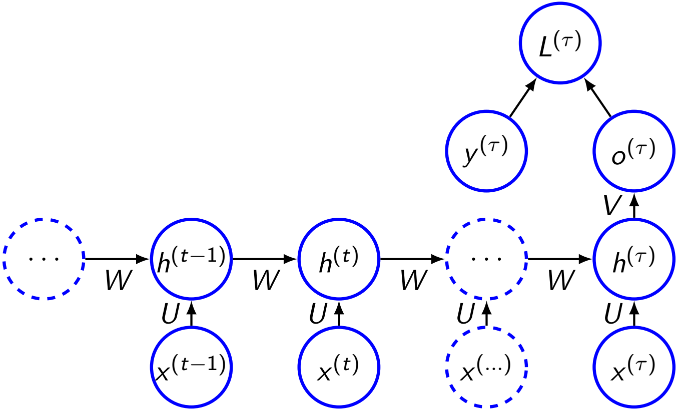
# Recurrent connection in hidden units



# Output to hidden unit connection



# Sequence processing

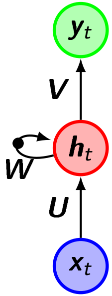


# Recurrent neural network

- Function computable by a Turing machine can be computed by such recurrent network of finite size
- Hyperbolic tangent is usually chosen as activation function for hidden units
- Output can considered as discrete, so $\boldsymbol{o}$ gives unnormalized log probabilities
- Forward propagation begins with initial state $\boldsymbol{h}^0$
- So we have,
  - $\boldsymbol{a}^{(t)} = \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}$
  - $\boldsymbol{h}^{(t)} = \tanh(\boldsymbol{a}^{(t)})$
  - $\boldsymbol{o}^{(t)} = \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}^{(t)}$
  - $\hat{\boldsymbol{y}}^{(t)} = \text{softmax}(\boldsymbol{o}^{(t)})$
- Input and output have the same length

# Backpropagation through time

- The network will be unfolded and gradient will be back propagated
- Number of stages need to be decided
- Issue in gradient computation
  - Vanishing gradients
  - Exploding gradients



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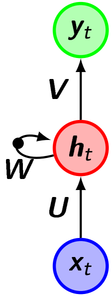
- Loss function
  - $E_t = \frac{1}{2}\sum_{k=1}^{\text{out}}(\hat{y}_k - y_k)^2$,



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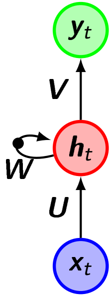
  - $E_t = \frac{1}{2}\sum_{k=1}^{\text{out}}(\hat{y}_k - y_k)^2$, $E = \frac{1}{2}\sum_{t=1}^{\tau}\sum_{k=1}^{\text{out}}(\hat{y}_{tk} - y_{tk})^2$



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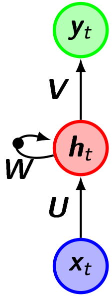
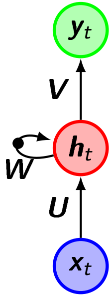
  - $E = -\sum_{t=1}^{\tau}\sum_{k=1}^{\text{out}}[\hat{y}_{tk}\ln y_{tk} + (1-\hat{y}_{tk})\ln(1-y_{tk})]$



# Backpropagation through time

- Basic equations

$$\boldsymbol{h}_t = \boldsymbol{U}\boldsymbol{x}_t + \boldsymbol{W}\phi(\boldsymbol{h}_{t-1})$$
$$\boldsymbol{y}_t = \boldsymbol{V}\phi(\boldsymbol{h}_t)$$



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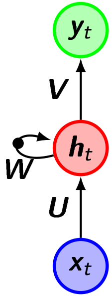
- Gradient

$$\frac{\partial E}{\partial W}$$



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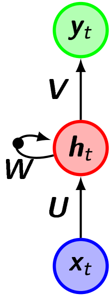
$$\frac{\partial E}{\partial \boldsymbol{W}} = \sum_{t=1}^{\tau} \frac{\partial E_t}{\partial \boldsymbol{W}}$$



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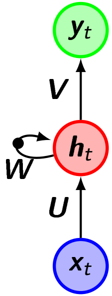

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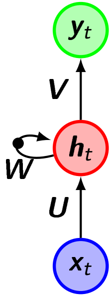

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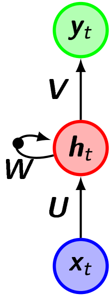

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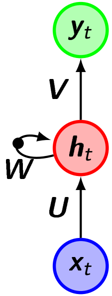

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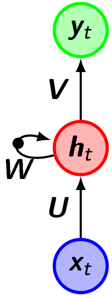
- Now we have,

$$\frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_k}$$



# Backpropagation through time

- Basic equations

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\end{aligned}
$$

- Gradient

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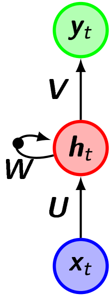
$$

- Now we have,

$$
\frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_k} = \prod_{i=k+1}^{t} \frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{h}_{i-1}}
$$



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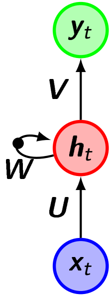
$$\frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_k} = \prod_{i=k+1}^{t} \frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{h}_{i-1}} = \prod_{i=k+1}^{t} \boldsymbol{W}^T \text{diag}[\phi'(\boldsymbol{h}_{i-1})]$$

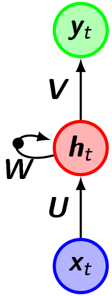


# Backpropagation through time

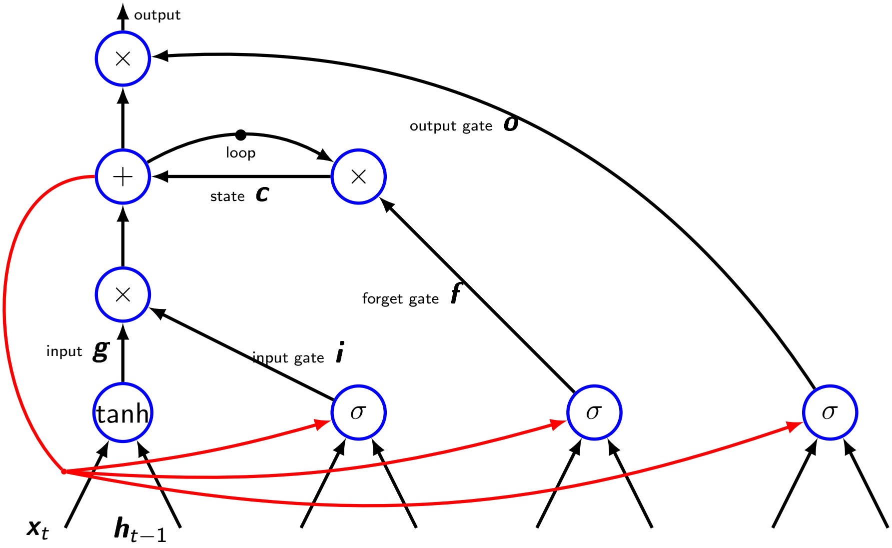
- Issues in gradient

$$\left\| \frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{h}_{i-1}} \right\| \leq \left\| \boldsymbol{W}^T \right\| \left\| \mathrm{diag}[\phi'(\boldsymbol{h}_{i-1})] \right\| \leq \lambda_{\boldsymbol{W}} \lambda_{\phi}$$

$$\left\| \frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_k} \right\| \leq (\lambda_{\boldsymbol{W}} \lambda_{\phi})^{t-k}$$



# LSTM


output
×
output gate $\boldsymbol{o}$
loop
+
×
state $\boldsymbol{c}$
×
forget gate $\boldsymbol{f}$
input $\boldsymbol{g}$
input gate $\boldsymbol{i}$
tanh
σ
σ
σ
$\boldsymbol{x}_t$
$\boldsymbol{h}_{t-1}$

# LSTM

- Mathematical relation

$$\boldsymbol{i}_t = \sigma(\boldsymbol{\theta}_{xi}\boldsymbol{x}_t + \boldsymbol{\theta}_{hi}\boldsymbol{h}_{t-1} + \boldsymbol{b}_i)$$

$$\boldsymbol{f}_t = \sigma(\boldsymbol{\theta}_{xf}\boldsymbol{x}_t + \boldsymbol{\theta}_{hf}\boldsymbol{h}_{t-1} + \boldsymbol{b}_f)$$

$$\boldsymbol{o}_t = \sigma(\boldsymbol{\theta}_{xo}\boldsymbol{x}_t + \boldsymbol{\theta}_{ho}\boldsymbol{h}_{t-1} + \boldsymbol{b}_o)$$

$$\boldsymbol{g}_t = \tanh(\boldsymbol{\theta}_{xg}\boldsymbol{x}_t + \boldsymbol{\theta}_{hg}\boldsymbol{h}_{t-1} + \boldsymbol{b}_g)$$

$$\boldsymbol{c}_t = \boldsymbol{f}_t \odot \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \odot \boldsymbol{g}_t$$

$$\boldsymbol{h}_t = \boldsymbol{o}_t \odot \tanh(\boldsymbol{c}_t)$$

# LSTM

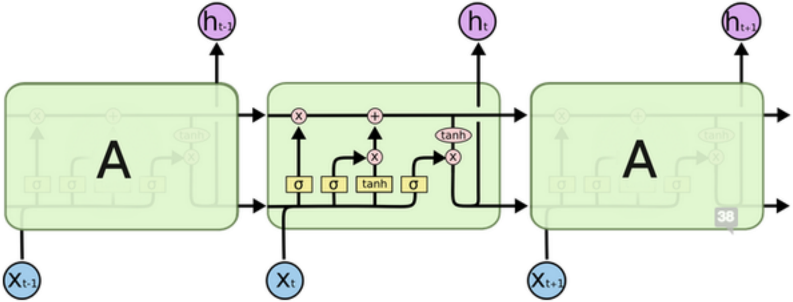



Image source:colah.github.io