Introduction to Deep Learning

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Neural Network

Human brain vs von Neumann computer

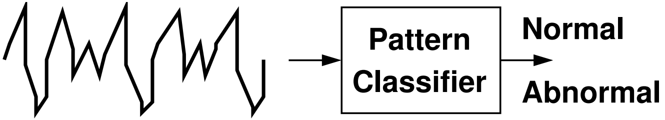
- Massive parallelism
- Distributed representation and computation
- Learning ability
- Generalization ability
- Adaptability
- Inherent contextual information processing
- Fault tolerance
- Low energy consumption

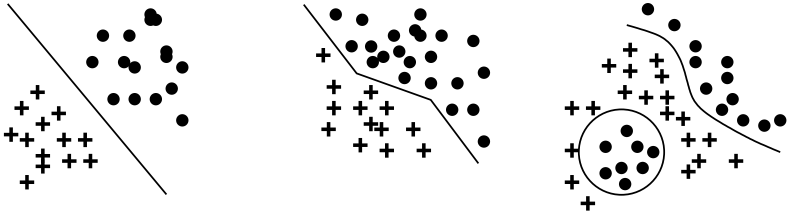
Computer vs Brain

	von Neumann	Neural system
Processor	Complex, high speed, one or a few	Simple, low speed, a large number
Memory	Separate from processor, Localized, Noncontent addressable	Integrated into processor, Distributed, Content addressable
Computing	Centralized, sequential, stored program	Distributed, parallel, self-learning
Reliability	Very vulnerable	Robust
Expertise	Numeric and symbolic manipulations	Perceptual problems
Operating environment	Well defined, well constrained	Poorly defined, unconstrained

Artificial Neuron: Applications

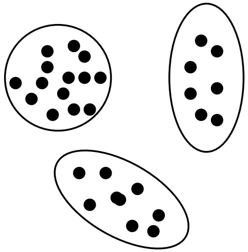
- Pattern classification





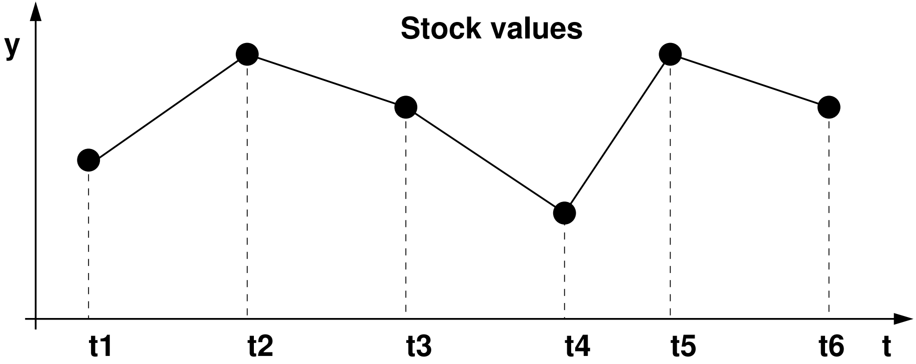
Artificial Neuron: Applications

- Clustering/categorization



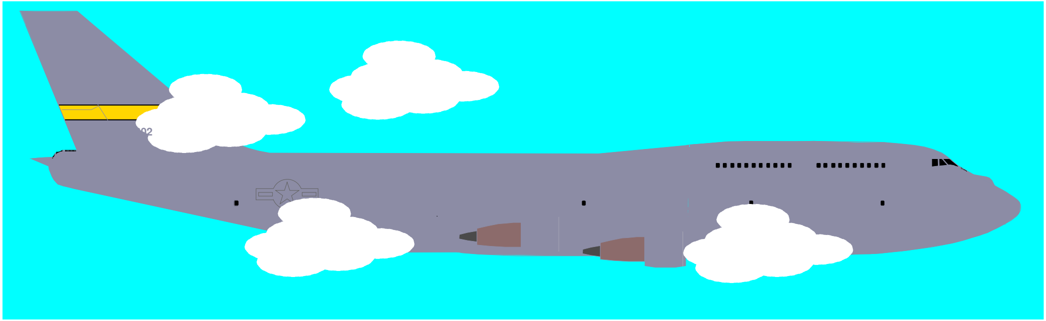
Artificial Neuron: Applications

- Prediction



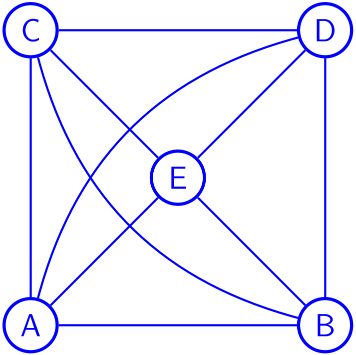
Artificial Neuron: Applications

- Retrieval



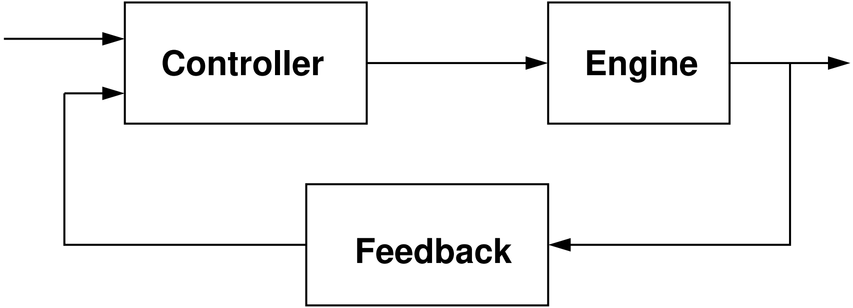
Artificial Neuron: Applications

- Optimization



Artificial Neuron

- Control



History

- Started in 1940s by McCulloch and Pitt
- Rosenblatt perceptron convergence theorem (1960)
- In 1980s ANN started gaining popularity
- Again became popular after 2006

Biological Neuron

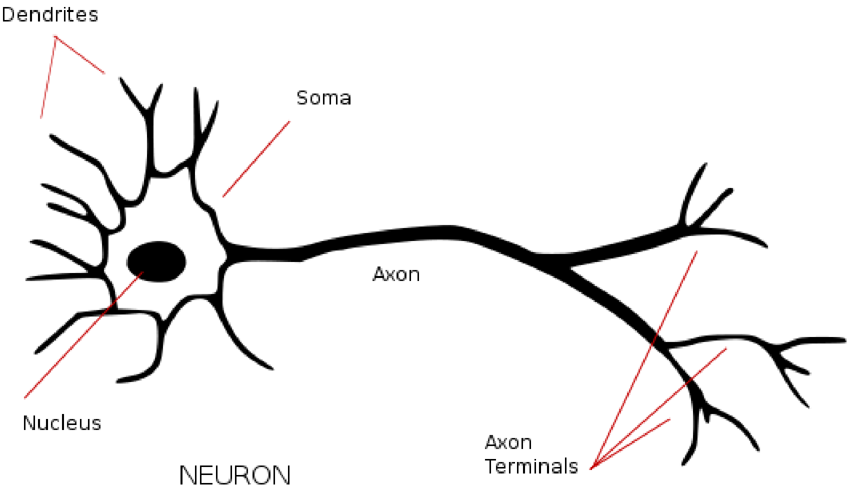


Image source: Internet

Cerebral cortex

- It is a flat sheet of neurons about 2-3 millimeter thick with surface area is 2200 cm^2
  - Twice the area of computer keyboard
- It contains around 10^{11} neurons
  - Number of stars in the Milky-way
- Each neuron is connected to 10^3-10^4 other neurons
- Total connections is around 10^{14}-10^{15}
- Connectionist model

Human brain

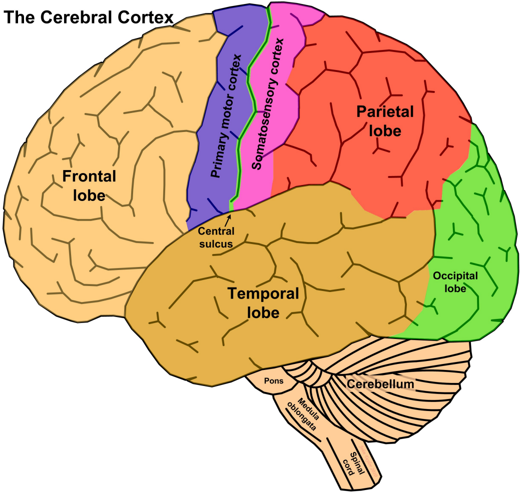
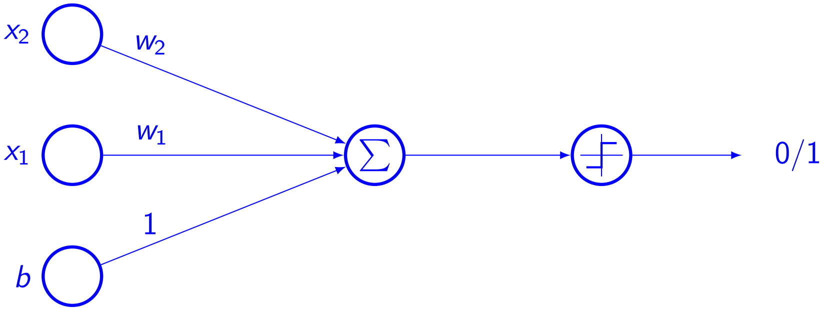


Image source: Internet

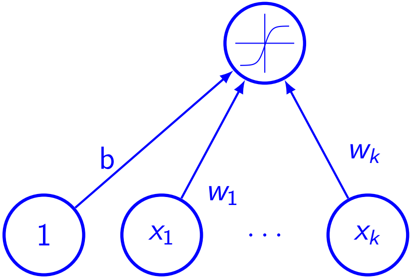
Neuron

- One of the primitive models

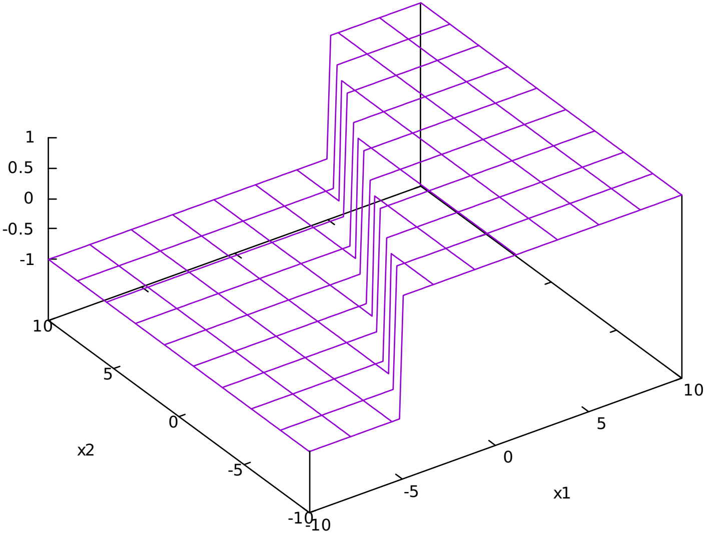


Artificial Neuron

- Neuron pre-activation function
  - $a(\boldsymbol{x}) = \sum_i w_i x_i + b = b + \boldsymbol{w}^T \boldsymbol{x}$
- Neuron output activation function
  - $h(\boldsymbol{x}) = g(a(\boldsymbol{x})) = g(\sum_i w_i x_i + b)$
- Notations
  - $\boldsymbol{w}$ — Weight vector
  - b — Neuron bias
  - $g(.)$ — Activation function

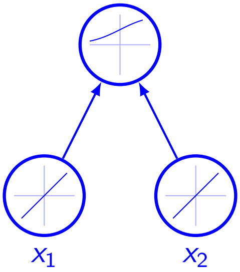


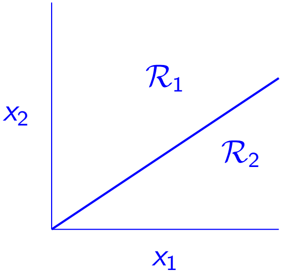
Physical interpretation



Classification using single neuron

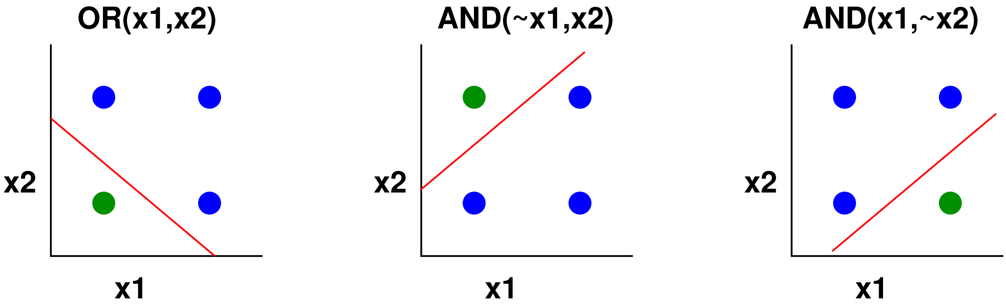
- Single neuron can do binary classification
  - Also known as logistic regression classifier





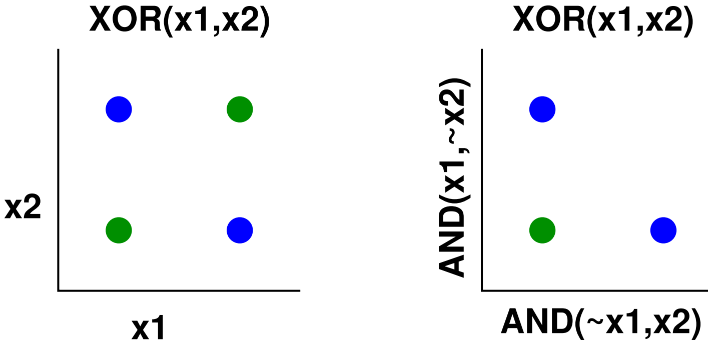
Artificial neuron

- Can solve linearly separable problems



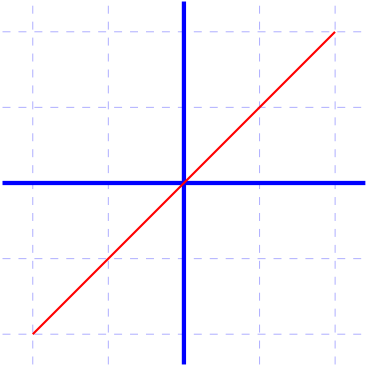
Artificial neuron: XOR problem

- There are issues for linear separation



Activation function

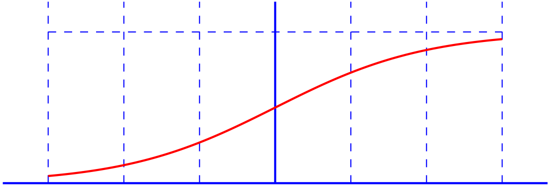
- Linear activation function
  - Not very interesting
  - No change in values
  - Huge range



$g(x) = x$

Activation function

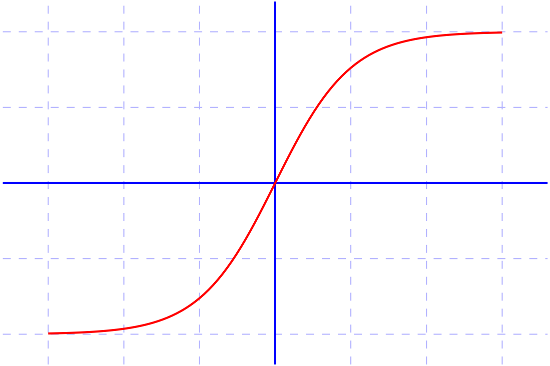
- Sigmoid function
  - Values lie between 0 and 1
  - Strictly increasing function
  - Bounded



$$g(x) = \mathrm{sigm}(x) = \frac{1}{1 + \exp(-x)}$$

Activation function

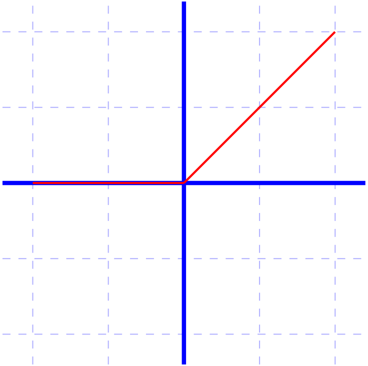
- Hyperbolic Tangent (Tanh) function
  - Can be positive or negative
  - Values lie between -1 and 1
  - Strictly increasing function
  - Bounded



$$g(x) = \tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

Activation function

- Rectified linear activation function
  - Bounded below by 0
  - Strictly increasing function
  - Not upper bounded



$$g(x) = \text{reclin}(x) = \max(0, x)$$

Single hidden layer neural network

- Hidden layer pre-activation

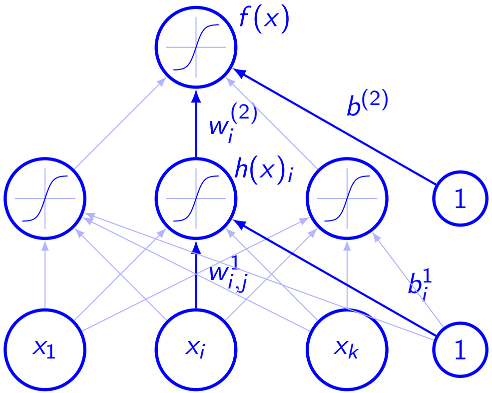
$$a(\mathbf{x}) = \mathbf{b}^1 + \mathbf{w}^1\mathbf{x}$$

- Hidden layer activation

$$h(\mathbf{x}) = g(a(\mathbf{x}))$$

- Output layer activation

$$f(\mathbf{x}) = o(b^{(2)} + \mathbf{w}^{(2)T} h^1(\mathbf{x}))$$



Multiclass classification

- Need multiple outputs that is one neuron for each class
- Need to determine probability of $p(y = c|\boldsymbol{x})$
- Softmax activation function is used at the output

$$\boldsymbol{o}(\boldsymbol{a}) = \text{softmax}(a) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \quad \frac{\exp(a_2)}{\sum_c \exp(a_c)} \quad \cdots \quad \frac{\exp(a_c)}{\sum_c \exp(a_c)} \right]^T$$

  - Strictly positive
  - Sum to 1
  - Class having the highest probability will be the predicted output

Multi layer neural network

- Pre-activation in layer $k > 0$ ($\boldsymbol{h}^{(0)}(\boldsymbol{x}) = x$)

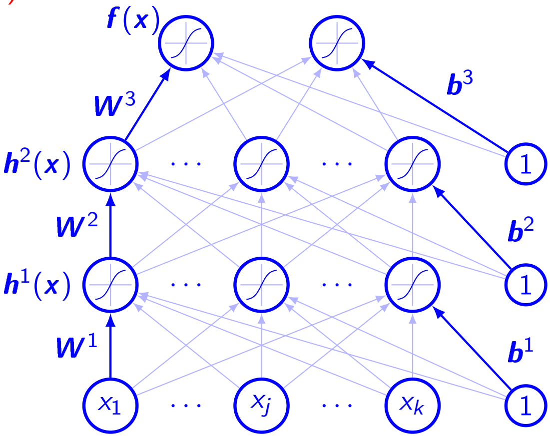
$$\boldsymbol{a}^{(k)}(\boldsymbol{x}) = \boldsymbol{b}^{(k)} + \boldsymbol{W}^{(k)}\boldsymbol{h}^{(k-1)}\boldsymbol{x}$$

- Hidden layer activation

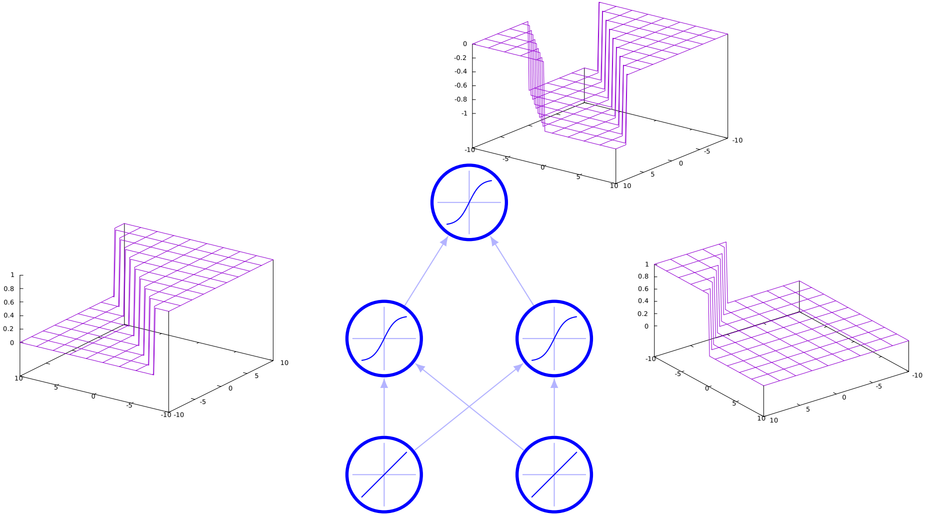
$$\boldsymbol{h}^{(k)}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{a}^{(k)}(\boldsymbol{x}))$$

- Output layer activation

$$\boldsymbol{h}^{(L+1)}(\boldsymbol{x}) = o(\boldsymbol{a}^{(L+1)}(\boldsymbol{x})) = \boldsymbol{f}(\boldsymbol{x})$$



Capacity of neural network



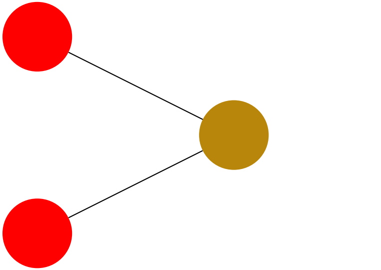
Capacity of neural network

- Universal approximation theorem (Hornik,1991)
  - A single hidden layer neural network with a linear output unit can approximate any continuous function arbitrarily well, given enough hidden units.
- The result is applicable for other hidden layer activation functions such as sigmoid, tanh, etc.
- This is a promising result, but it does not say that there is a learning algorithm to find the necessary parameter values!

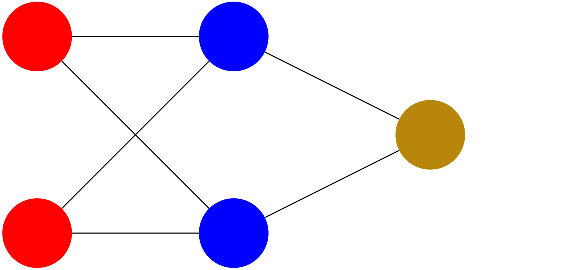
Types of Neural Network

- Feed forward neural network
- Radial basis function network
- Recurrent neural network
- Boltzmann machine
- Long short term memory network
- and many more

Perceptron

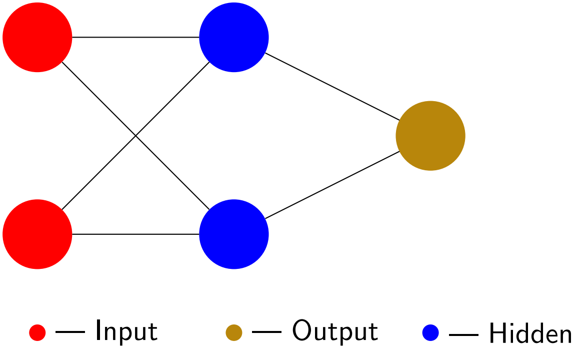


Feed Forward

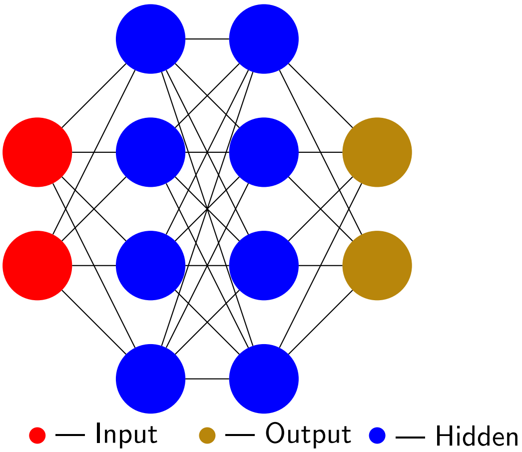


Radial Basis Function

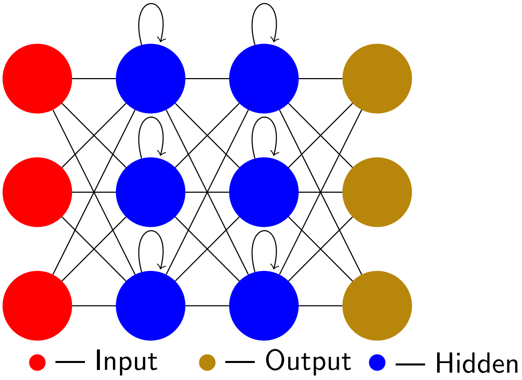
- Typically it will have 3 layers
- Distance from a center vector is computed



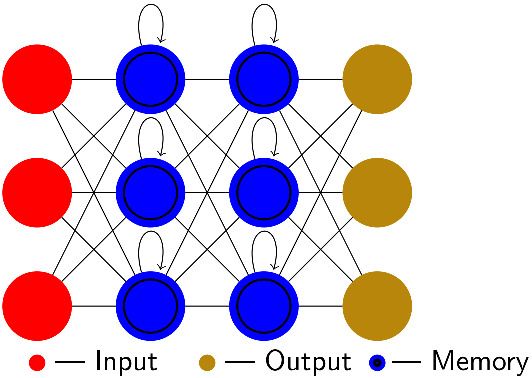
Deep Feed Forward



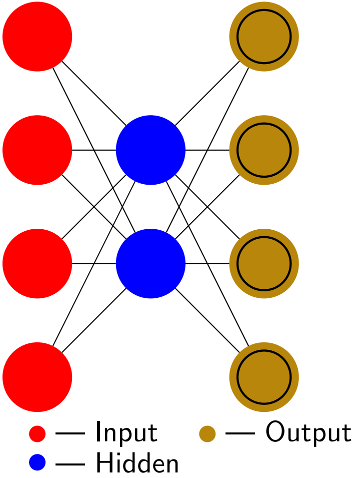
Recurrent Neural Network



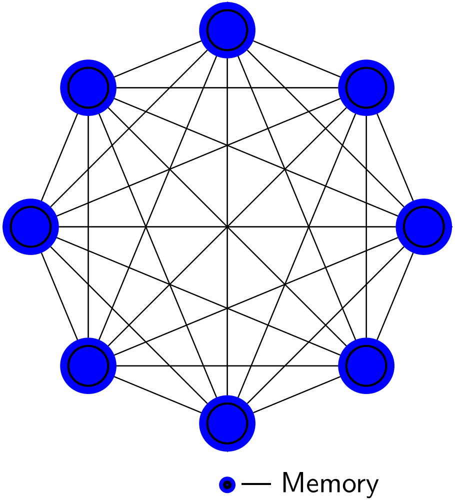
Long Short Term Memory



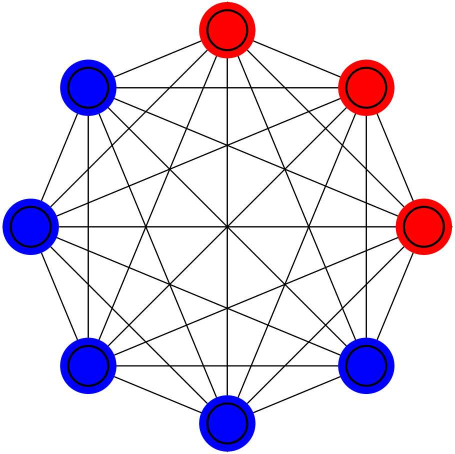
Auto Encoder



Markov chain



Boltzmann Machine



● — Input ● — Hidden

Learning the parameters

- The network must learn the connection weights from available training examples
- Learning can be
  - Supervised
  - Unsupervised
  - Hybrid
- Four basic types of learning rule
  - Error correction rule
  - Boltzmann learning
  - Hebbian
  - Competitive learning

Error correction rule

- Output is generated based on the weight values but this may vary from desired value
- The error information is used to update the weight value
- Perceptron learning algorithm
  - Initialize the weights and threshold to small random numbers
  - Present a pattern vector and evaluate the output of neuron
  - Update the weight according to $w_j(t+1) = w_j(t) + \eta(d-y)x_j$
- Back propagation algorithm

Boltzmann learning

- Usually symmetric recurrent network consisting of binary units
- A subset of neurons interact with environment
- Generally it has two modes
  - Clamped — Visible neurons are clamped to specific states
  - Free-running - Visible and hidden unit operate freely
- Stochastic learning rule derived from information theoretic and thermodynamic principles
- Learning rule is given by $\Delta w_{ij} = \eta(\bar{\rho}_{ij} - \rho_{ij})$

Hebbian rule

- One of the oldest learning rules
- If neuron on both sides of a synapse are activated synchronously and repeatedly, the synapse's strength is selectively increased
- Mathematically, it can be described as $w_{ij}(t+1) = w_{ij}(t) + \eta y_j(t) x_i(t)$

Competitive learning rule

- Output units compete among themselves for activation
- Only one output is active at time
- Also known as winner-take-all
- Mathematically, it can be represented as $\mathbf{w}_{i^*}\mathbf{x} \geq \mathbf{w}_i\mathbf{x}$
- Competitive learning rule can be stated as

$$\Delta w_{ij} = \begin{cases} \eta(x_j^u - w_{i^*j}) & i = i^* \\ 0 & i \neq i^* \end{cases}$$

Summary

- Error correction rule — Single or multilayer perceptron
  - Pattern classification, function approximation, prediction, control
- Boltzmann — Recurrent
  - Pattern classification
- Hebbian — Multilayer feed forward
  - Pattern classification, data analysis
- Competitive
  - Within class categorization, data compression